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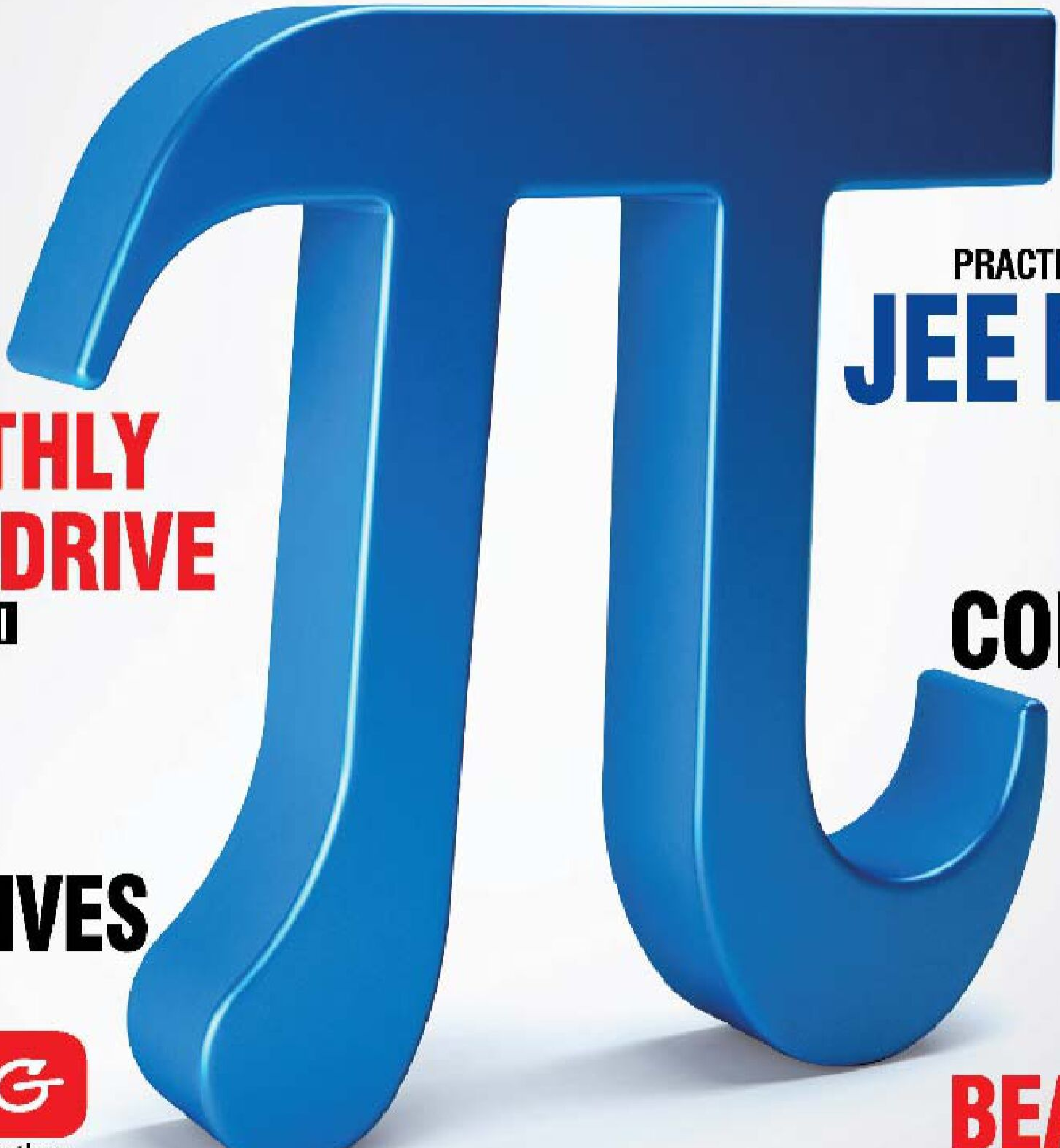

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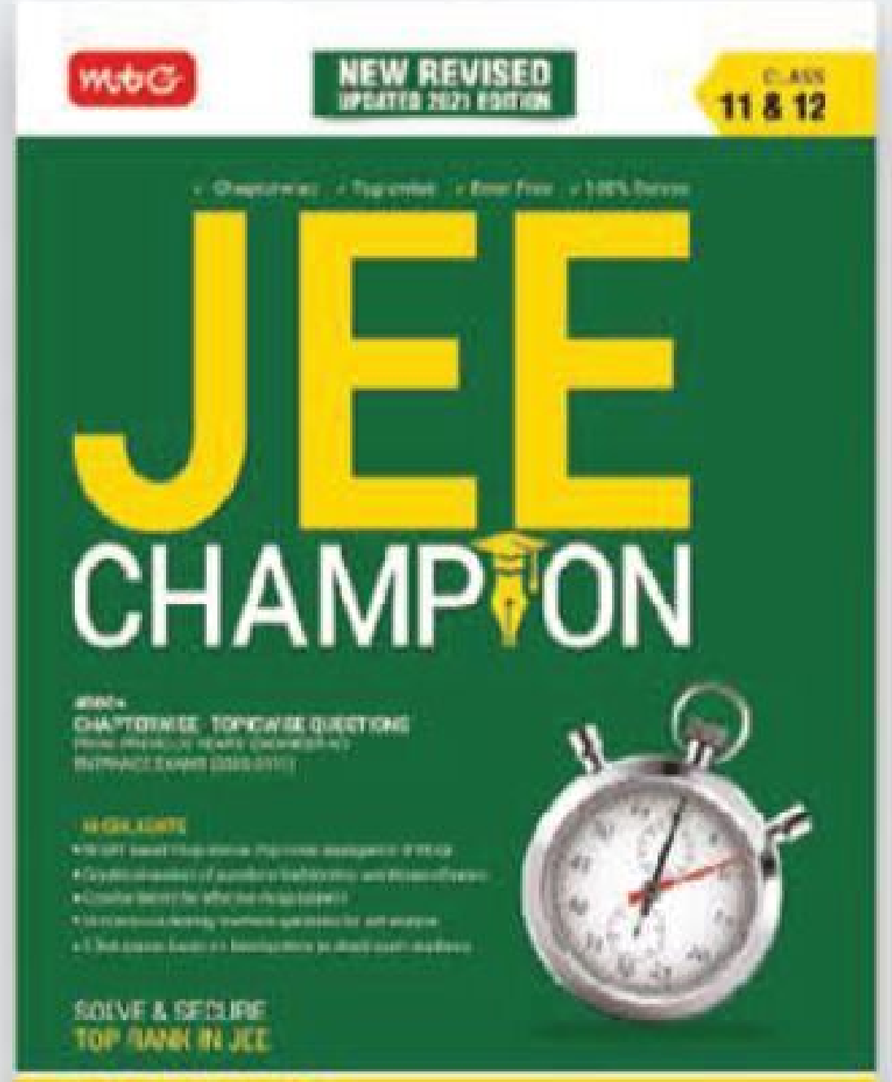
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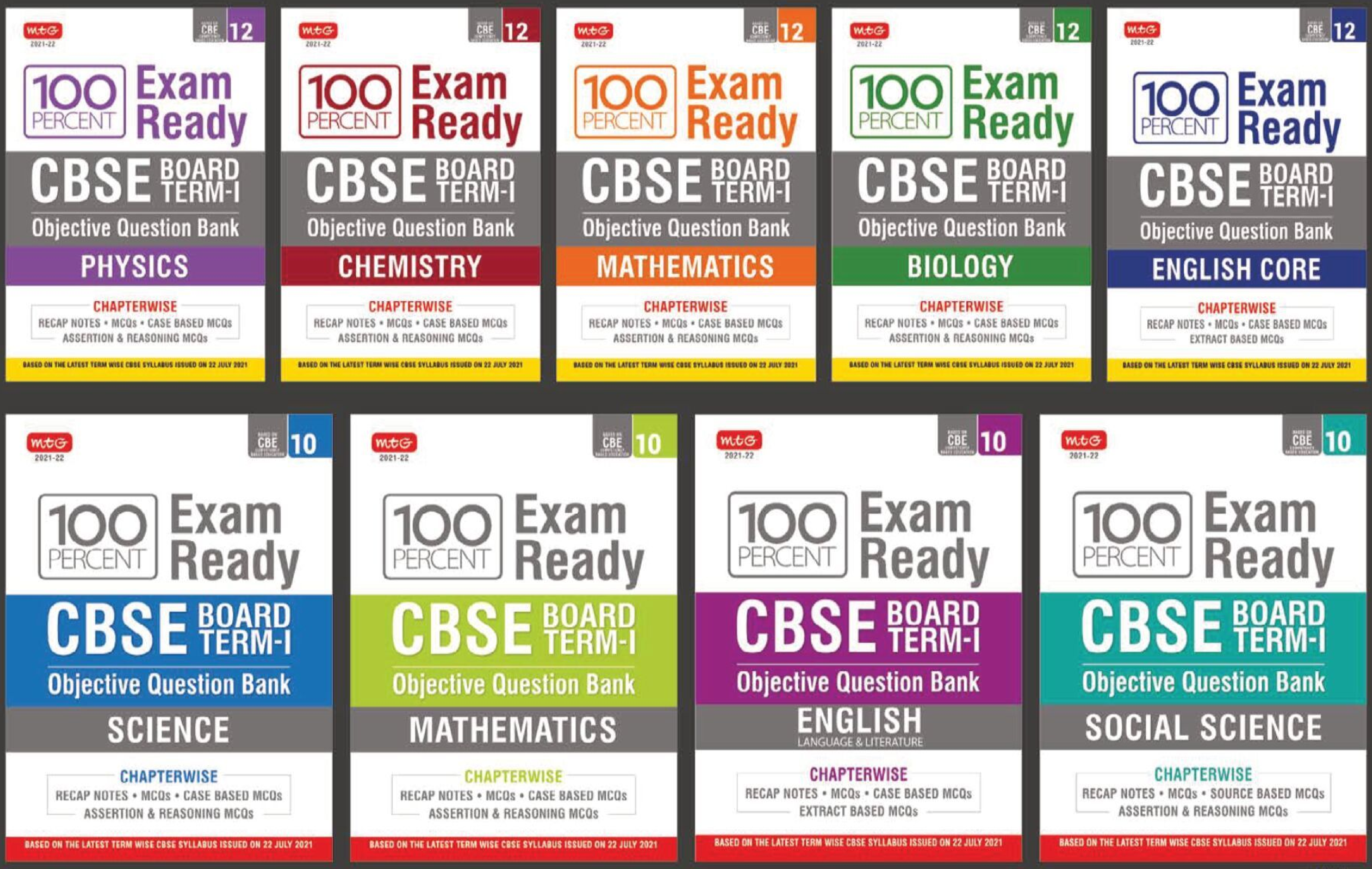
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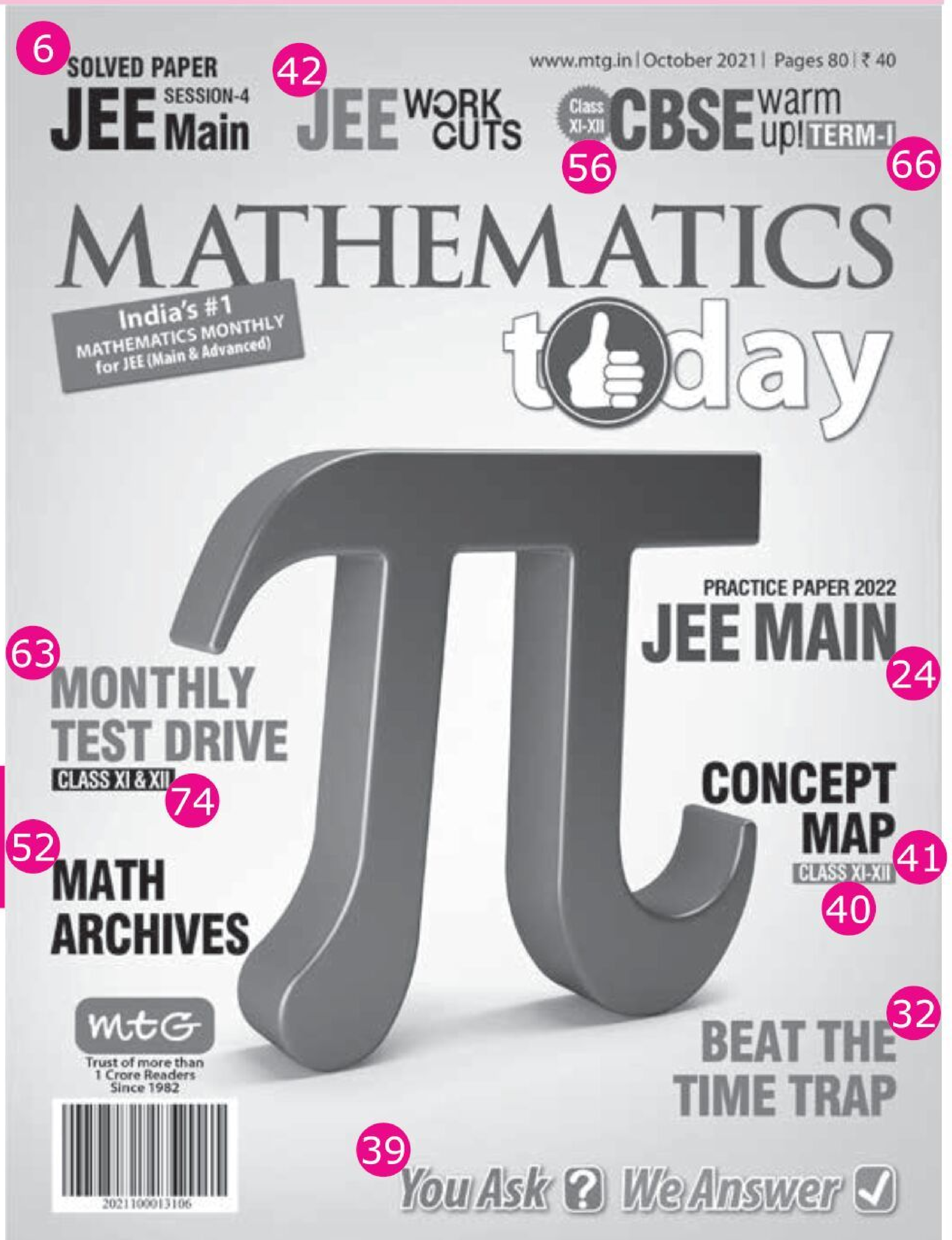
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SOLVED PAPER

Held on
26th August, Morning
Shift

JEE MAIN 2021

SECTION-A (MULTIPLE CHOICE QUESTIONS)

1. The value of $\lim_{n\to\infty} \frac{1}{n} \sum_{r=0}^{2n-1} \frac{n^2}{n^2+4r^2}$ is

(a) $\frac{1}{2}\tan^{-1}(2)$ (b) $\tan^{-1}(4)$

(c) $\frac{1}{2}\tan^{-1}(4)$ (d) $\frac{1}{4}\tan^{-1}(4)$

2. Let $\vec{a} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{b} = \hat{j} - \hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}\times\vec{c} = \vec{b}$ and $\vec{a}\cdot\vec{c} = 3$, then $\vec{a}\cdot(\vec{b}\times\vec{c})$ is equal to

(a) 6 (b) -2 (c) 2 (d) -6

3. Let A and B be independent events such that $P(A) = p$, $P(B) = 2p$. The largest value of p, for which P (exactly one of A, B occurs) = 5/9, is

(a) $\frac{4}{9}$ (b) $\frac{1}{3}$ (c) $\frac{5}{12}$ (d) $\frac{2}{9}$

4. On the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$, let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x + 2y = 0$. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $(5 - e^2)\cdot A$ is

(a) 12 (b) 6 (c) 14 (d) 24

5. The sum of solutions of the equation $\frac{\cos x}{1+\sin x} = |\tan 2x|$, $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is

(a) $\frac{\pi}{10}$ (b) $-\frac{7\pi}{30}$ (c) $-\frac{\pi}{15}$ (d) $-\frac{11\pi}{30}$

6. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set

(a) {79, 81, 83, 85} (b) {84, 87, 90, 93}

(c) {80, 83, 86, 89} (d) {84, 86, 88, 90}

7. A plane P contains the line $x + 2y + 3z + 1 = 0 = x - y - z - 6$, and is perpendicular to the plane $-2x + y + z + 8 = 0$. Then which of the following points lie on P?

(a) (2, −1, 1) (b) (0, 1, 1)

(c) (−1, 1, 2) (d) (1, 0, 1)

8. Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations

$(1 + \cos^2\theta)x + \sin^2\theta\, y + 4\sin 3\theta\, z = 0$
$\cos^2\theta\, x + (1 + \sin^2\theta)\, y + 4\sin 3\theta\, z = 0$
$\cos^2\theta\, x + \sin^2\theta\, y + (1 + 4\sin 3\theta)\, z = 0$

has a non-trivial solution, then the value of θ is

(a) $\frac{4\pi}{9}$ (b) $\frac{5\pi}{18}$ (c) $\frac{7\pi}{18}$ (d) $\frac{\pi}{18}$

9. The sum of the series

$\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + \ldots + \frac{2^{100}}{x^{2^{100}}+1}$, when $x = 2$, is

(a) $1 - \frac{2^{100}}{4^{100}-1}$ (b) $1 + \frac{2^{101}}{4^{101}-1}$

(c) $1 + \frac{2^{100}}{4^{101}-1}$ (d) $1 - \frac{2^{101}}{4^{101}-1}$

10. If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of $(1 + x)^{20}$, then the value of $\sum_{r=0}^{20} r^2\, {}^{20}C_r$ is equal to

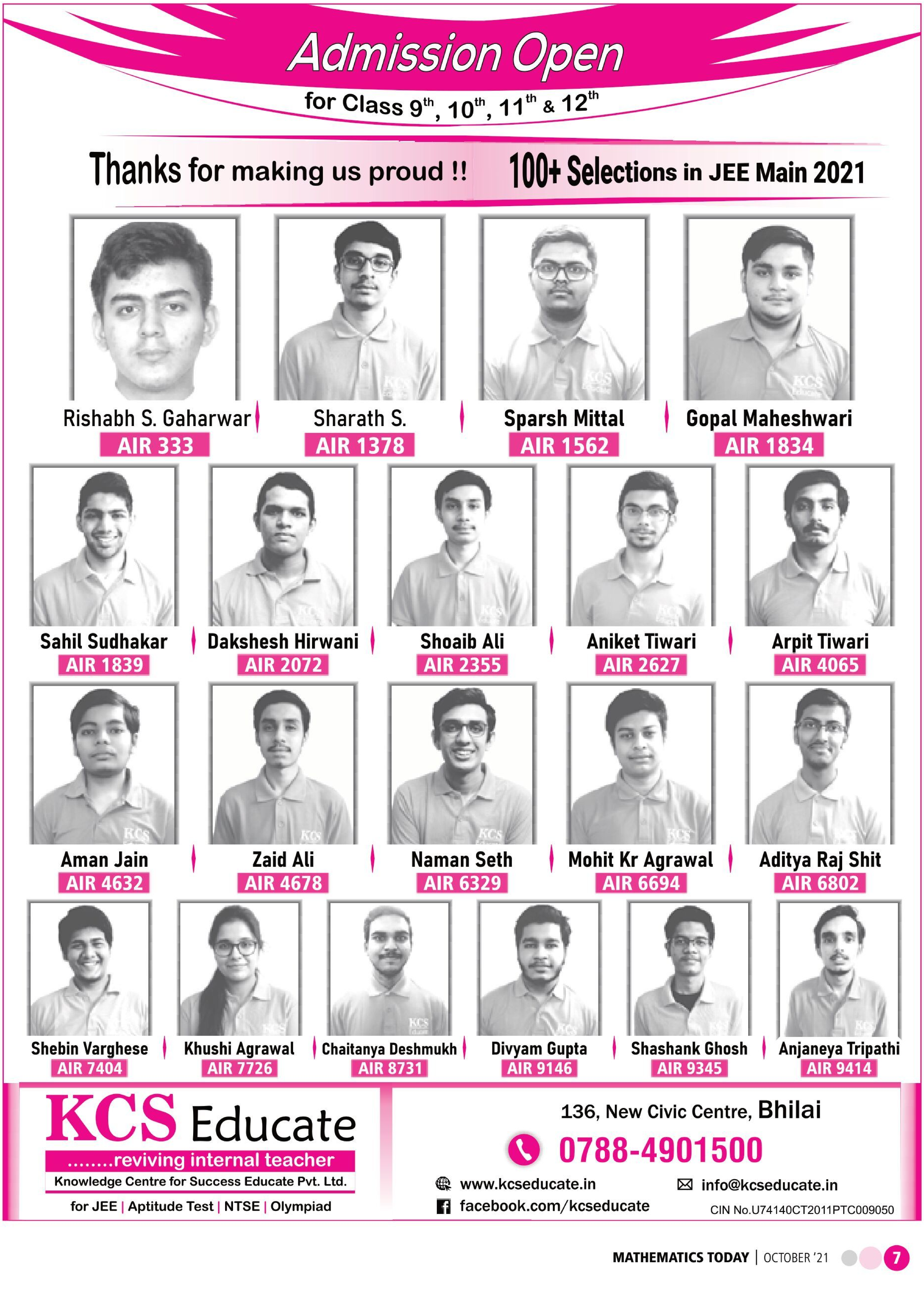
(a) 420×2^{18} (b) 380×2^{18}

(c) 380×2^{19} (d) 420×2^{19}

11. If the truth value of the Boolean expression $((p \vee q) \wedge (q \to r) \wedge (\sim r)) \to (p \wedge q)$ is false, then the truth values of the statements p, q, r respectively can be

(a) F T F (b) T F F

(c) T F T (d) F F T



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12. Let $f(x)=\cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$, $0 < x < 1$. Then

(a) $(1 + x)^2 f'(x) + 2(f(x))^2 = 0$
(b) $(1 - x)^2 f'(x) + 2(f(x))^2 = 0$
(c) $(1 - x)^2 f'(x) - 2(f(x))^2 = 0$
(d) $(1 + x)^2 f'(x) - 2(f(x))^2 = 0$

13. If $A=\begin{pmatrix}\frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}}\\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}}\end{pmatrix}$, $B=\begin{pmatrix}1 & 0\\ i & 1\end{pmatrix}$, $i=\sqrt{-1}$ and $Q=A^TBA$, then inverse of the matrix $AQ^{2021}A^T$ is equal to

(a) $\begin{pmatrix}1 & -2021i\\ 0 & 1\end{pmatrix}$ (b) $\begin{pmatrix}1 & 0\\ -2021i & 1\end{pmatrix}$

(c) $\begin{pmatrix}\frac{1}{\sqrt{5}} & -2021\\ 2021 & \frac{1}{\sqrt{5}}\end{pmatrix}$ (d) $\begin{pmatrix}1 & 0\\ 2021i & 1\end{pmatrix}$

14. Let ABC be a triangle with $A(-3, 1)$ and $\angle ACB = \theta$, $0 < \theta < \pi/2$. If the equation of the median through B is $2x + y - 3 = 0$ and the equation of angle bisector of C is $7x - 4y - 1 = 0$, then $\tan\theta$ is equal to

(a) 3/4 (b) 4/3 (c) 2 (d) 1/2

15. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is

(a) (10.5, 26) (b) (10.5, 25)
(c) (11, 25) (d) (11, 26)

16. The equation $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ represents a circle with

(a) centre at (0, 0) and radius $\sqrt{2}$
(b) centre at (0, 1) and radius 2
(c) centre at (0, −1) and radius $\sqrt{2}$
(d) centre at (0, 1) and radius $\sqrt{2}$

17. If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point $(-30, 0)$ and is tangent to the parabola $y^2 = 30x$, then the length of this chord is

(a) 5 (b) 7 (c) $3\sqrt{5}$ (d) $5\sqrt{3}$

18. The value of

$$\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left(\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right)^{1/2}dx \text{ is}$$

(a) $\log_e 4$ (b) $2\log_e 16$
(c) $\log_e 16$ (d) $4\log_e(3+2\sqrt{2})$

19. If the sum of an infinite GP $a, ar, ar^2, ar^3, \ldots$ is 15, and the sum of the squares of its each term is 150, then the sum of $ar^2, ar^4, ar^6, \ldots$ is

(a) $\frac{25}{2}$ (b) $\frac{9}{2}$ (c) $\frac{1}{2}$ (d) $\frac{5}{2}$

20. Let $y = y(x)$ be a solution curve of the differential equation $(y+1)\tan^2 x\,dx+\tan x\,dy+y\,dx=0$, $x\in\left(0, \frac{\pi}{2}\right)$. If $\lim_{x\to 0^+} xy(x)=1$, then the value of $y(\pi/4)$ is

(a) $\frac{\pi}{4}+1$ (b) $\frac{\pi}{4}-1$ (c) $\frac{\pi}{4}$ (d) $-\frac{\pi}{4}$

SECTION-B (NUMERICAL VALUE TYPE)

Attempt any 5 questions out of 10.

1. If ${}^1P_1 + 2\cdot{}^2P_2 + 3\ {}^3P_3 + \cdots + 15\cdot{}^{15}P_{15} = {}^qP_r - s'$ $0 \le s \le 1$, then ${}^{q+s}C_{r-s}$ is equal to ______.

2. Let the line L be the projection of the line $\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$ in the plane $x - 2y - z = 3$. If d is the distance of the point $(0, 0, 6)$ from L, then d^2 is equal to ______.

3. The area of the region $S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ is ______.

4. The locus of a point, which moves such that the sum of squares of its distances from the points (0, 0), (1, 0), (0, 1), (1, 1) is 18 units, is a circle of diameter d. Then d^2 is equal to ______.

5. The sum of all integral values of k $(k \ne 0)$ for which the equation $\frac{2}{x-1}-\frac{1}{x-2}=\frac{2}{k}$ in x has no real roots, is ______.

6. Let $a, b \in R$, $b \ne 0$. Define a function

$$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x-1), & \text{for } x\le 0\\ \frac{\tan 2x-\sin 2x}{bx^3}, & \text{for } x>0\end{cases}$$

If f is continuous at $x = 0$, then $10 - ab$ is equal to ______.

7. A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum, and the circumference of the circle is k (meter), then $\left(\frac{4}{\pi}+1\right)k$ is equal to ______.

8. Let $z=\frac{1-i\sqrt{3}}{2}$, $i=\sqrt{-1}$. Then the value of

$$21+\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3+\ldots+\left(z^{21}+\frac{1}{z^{21}}\right)^3$$ is ________.

9. The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is ________.

10. If $y = y(x)$ is an implicit function of x such that $\log_e(x+y) = 4xy$, then $\frac{d^2y}{dx^2}$ at $x = 0$ is equal to _______.

SOLUTIONS

1. (c) : We have, $\lim\limits_{n\to\infty} \times \frac{1}{n}\sum\limits_{r=0}^{2n-1}\frac{n^2}{(n^2+4r^2)}$

$$=\lim_{n\to\infty}\frac{1}{n}\sum_{r=0}^{2n-1}\frac{1}{1+4\left(\frac{r}{n}\right)^2}=L \quad \text{(say)}$$

When $r = 0$, then $x = 0$ and when $r = 2n - 1$, then

$x=2-\frac{1}{n} \Rightarrow x\to 2$ as $n\to\infty$

$$L=\int_0^2\frac{1}{1+4x^2}\,dx=\frac{1}{4}\int_0^2\frac{dx}{\left(\frac{1}{2}\right)^2+x^2}$$

$$=\frac{1}{4}\cdot 2\left[\tan^{-1}\left(\frac{x}{\left(\frac{1}{2}\right)}\right)\right]_0^2$$

$$=\frac{1}{2}\left[\tan^{-1}2x\right]_0^2$$

$$=\frac{1}{2}\left[\tan^{-1}4-0\right]=\frac{\tan^{-1}4}{2}$$

2. (b) : Let $\vec{c}=c_1\hat{i}+c_2\hat{j}+c_3\hat{k}$

Given $\vec{a}\cdot\vec{c}=3=(\hat{i}+\hat{j}+\hat{k})\cdot(c_1\hat{i}+c_2\hat{j}+c_3\hat{k})$

$\Rightarrow \vec{a}\cdot\vec{c}=c_1+c_2+c_3=3$ ...(i)

Now, $\vec{a}\times\vec{c}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & 1 & 1\\ c_1 & c_2 & c_3\end{vmatrix}$

$=(c_3-c_2)\hat{i}+(c_1-c_3)\hat{j}+(c_2-c_1)\hat{k}=\hat{j}-\hat{k}$ (Given)

$\therefore$ $c_3 = c_2$, $c_1 - c_3 = 1$, $c_1 - c_2 = 1$ ...(ii)

Substituting $c_2 = c_3$ in (i)

$c_1 + 2c_2 = 3$ ...(iii)

Solving (ii) and (iii), we get

$c_2=\frac{2}{3},\ c_3=\frac{2}{3},\ c_1=\frac{5}{3}$

$\Rightarrow \vec{c}=\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+\frac{2}{3}\hat{k}$

Now, $\vec{a}\cdot(\vec{b}\times\vec{c})=\begin{vmatrix}1 & 1 & 1\\ 0 & 1 & -1\\ \frac{5}{3} & \frac{2}{3} & \frac{2}{3}\end{vmatrix}$

$=\frac{4}{3}-\frac{5}{3}-\frac{5}{3}=-2$

3. (c) : We have, $P(A) = p$ and $P(B) = 2p$

Probability that exactly one of A, B occurs

$=P(A\cap\overline{B})+P(\overline{A}\cap B)=\frac{5}{9}$

$\Rightarrow P(A)\cdot P(\overline{B})+P(\overline{A})P(B)=\frac{5}{9}$

$\Rightarrow P(A)(1-P(B))+(1-P(A))P(B)=\frac{5}{9}$

$\Rightarrow P(A)+P(B)-2P(A)\cdot P(B)=\frac{5}{9}$

$\Rightarrow p+2p-2\times 2p^2=\frac{5}{9} \Rightarrow 3p-4p^2=\frac{5}{9}$

$\Rightarrow 36p^2 - 12p - 15p + 5 = 0$

$\Rightarrow 12p(3p - 1) - 5(3p - 1) = 0$

$\Rightarrow p=\frac{1}{3}$ or $\frac{5}{12}$

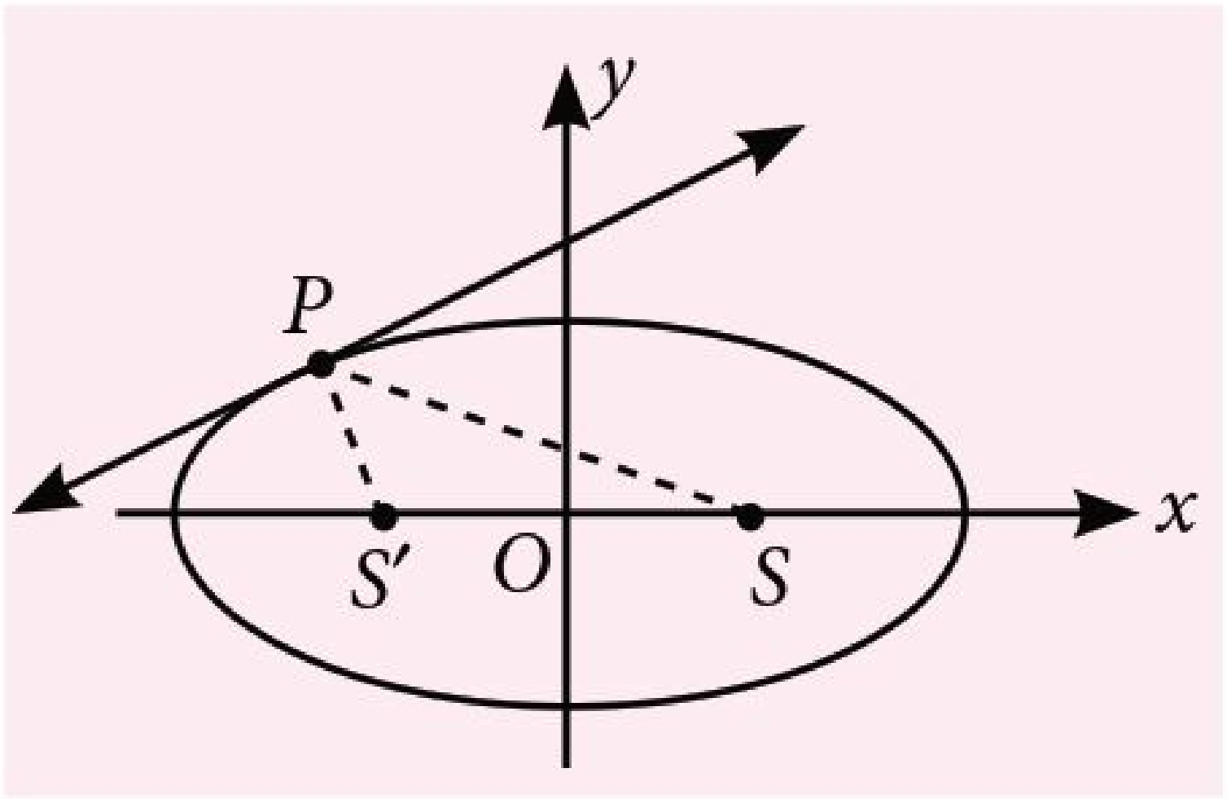
Larger value of p is $\frac{5}{12}$.

4. (b) : $\frac{x^2}{8}+\frac{y^2}{4}=1$

Since, $b^2 = a^2(1 - e^2)$

$\therefore$ $4 = 8(1 - e^2)$

$\Rightarrow e=\frac{1}{\sqrt{2}}$



Also, slope of tangent at $P = 2 = m$ (say)

$$\therefore \quad \text{Coordinates of } P \equiv \left(\frac{-a^2m}{\sqrt{a^2m^2+b^2}}, \frac{b^2}{\sqrt{a^2m^2+b^2}}\right)$$

(∵ P lies in second quadrant)

$$= \left(\frac{-8\times 2}{\sqrt{8\times 4+4}}, \frac{4}{\sqrt{8\times 4+4}}\right) = \left(\frac{-8}{3}, \frac{2}{3}\right)$$

Now, $ae = 2\sqrt{2}\times\frac{1}{\sqrt{2}} = 2$

∴ $SS' = 2ae = 4$

Area of triangle SPS', $A = \frac{1}{2}\times 4\times\frac{2}{3} = \frac{4}{3}$ sq. units

$$\therefore\ (5-e^2)A = \left(5-\frac{1}{2}\right)\times\frac{4}{3} = \frac{9}{2}\times\frac{4}{3} = 6$$

5. (d) : We have,

$$\frac{\cos x}{1+\sin x} = |\tan 2x|,\ x\in\left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$$

Case I : If $0 < x < \frac{\pi}{4}$ or $-\frac{\pi}{2} < x < -\frac{\pi}{4}$

In this case, $\tan 2x > 0$

Now, we have $\frac{\cos x}{1+\sin x} = \frac{2\tan x}{1-\tan^2 x}$

$$\Rightarrow \quad \frac{\cos x}{1+\sin x} = \frac{2\sin x\cos x}{\cos^2 x-\sin^2 x}$$

$\Rightarrow \quad \cos^2 x - \sin^2 x = 2\sin x + 2\sin^2 x$

[∵ $\cos x \neq 0$ in this case]

$\Rightarrow \quad 4\sin^2 x + 2\sin x - 1 = 0$

$$\therefore \quad \sin x = \frac{-2\pm\sqrt{4+16}}{8} = \frac{-2\pm 2\sqrt{5}}{8} = \frac{-1\pm\sqrt{5}}{4}$$

$$\Rightarrow \quad \sin x = \frac{\sqrt{5}-1}{4} \text{ or } -\frac{(\sqrt{5}+1)}{4}$$

$$\Rightarrow \quad x = \frac{\pi}{10} \text{ or } \frac{-3\pi}{10}$$

Case II : If $\frac{\pi}{4} < x < \frac{\pi}{2}$ or $-\frac{\pi}{4} < x < 0$

In this case, $\tan 2x < 0$

Now, we have $\frac{\cos x}{1+\sin x} = \frac{-2\sin x\cos x}{\cos^2 x-\sin^2 x}$

$\Rightarrow \quad 1 - 2\sin^2 x = -2\sin x - 2\sin^2 x$

$\Rightarrow \quad \sin x = -\frac{1}{2}$

$\Rightarrow \quad x = -\frac{\pi}{6}$

So, sum of solutions $= \frac{\pi}{10} - \frac{3\pi}{10} - \frac{\pi}{6}$

$$= \frac{3\pi-9\pi-5\pi}{30} = -\frac{11\pi}{30}$$

6. (a) : Let total number of patients in the hospital be 100 and also let $n(A)$ be the number of patients suffering from heart ailment, $n(B)$ be the number of patients from suffering lung infection.

Then, $n(A \cap B)$ be the number of patients suffering from both ailments.

Also, $n(A) = 89$

and $n(B) = 98$

∵ $\quad n(A\cap B) \le n(A)$ and $n(A\cap B) \le n(B)$

∴ $\quad n(A\cap B) \le 89$ ...(i)

Now, as $n(A) + n(B) - n(A\cap B) = n(A\cup B) \le 100$

∴ $\quad 89 + 98 - n(A\cap B) \le 100$

$\Rightarrow \quad n(A\cap B) \ge 87$ ...(ii)

From (i) and (ii), we get

$87 \le n(A\cap B) \le 89$

or $\quad 87 \le K \le 89$

∴ $\quad$ No element of set given in option (a) satisfies this condition.

7. (b) : Equation of plane containing the given lines is

$x + 2y + 3z + 1 + \lambda(x - y - z - 6) = 0$

$\Rightarrow \quad (1+\lambda)x + (2-\lambda)y + (3-\lambda)z + 1 - 6\lambda = 0$

This plane is ⊥ to plane $-2x + y + z + 8 = 0$

$\Rightarrow \quad -2(1+\lambda) + (1)(2-\lambda) + (1)(3-\lambda) = 0$

$$\Rightarrow \quad 3-4\lambda = 0 \Rightarrow \lambda = \frac{3}{4}$$

∴ $\quad$ Equation of the plane is

$$\frac{7}{4}x + \frac{5}{4}y + \frac{9}{4}z - \frac{7}{2} = 0$$

$\Rightarrow \quad 7x + 5y + 9z - 14 = 0$

Only point $(0, 1, 1)$ satisfies this equation.

8. (c) : Condition to have non-trivial solution is $\Delta = 0$.

$$\Rightarrow \quad \begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

Applying $R_1 \to R_1 - R_2$ and then $R_2 \to R_2 - R_3$, we get

$$\begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$\Rightarrow \quad 1 + 4\sin 3\theta + \sin^2\theta + \cos^2\theta = 0$

$$\Rightarrow \quad \sin 3\theta = \frac{-1}{2}$$

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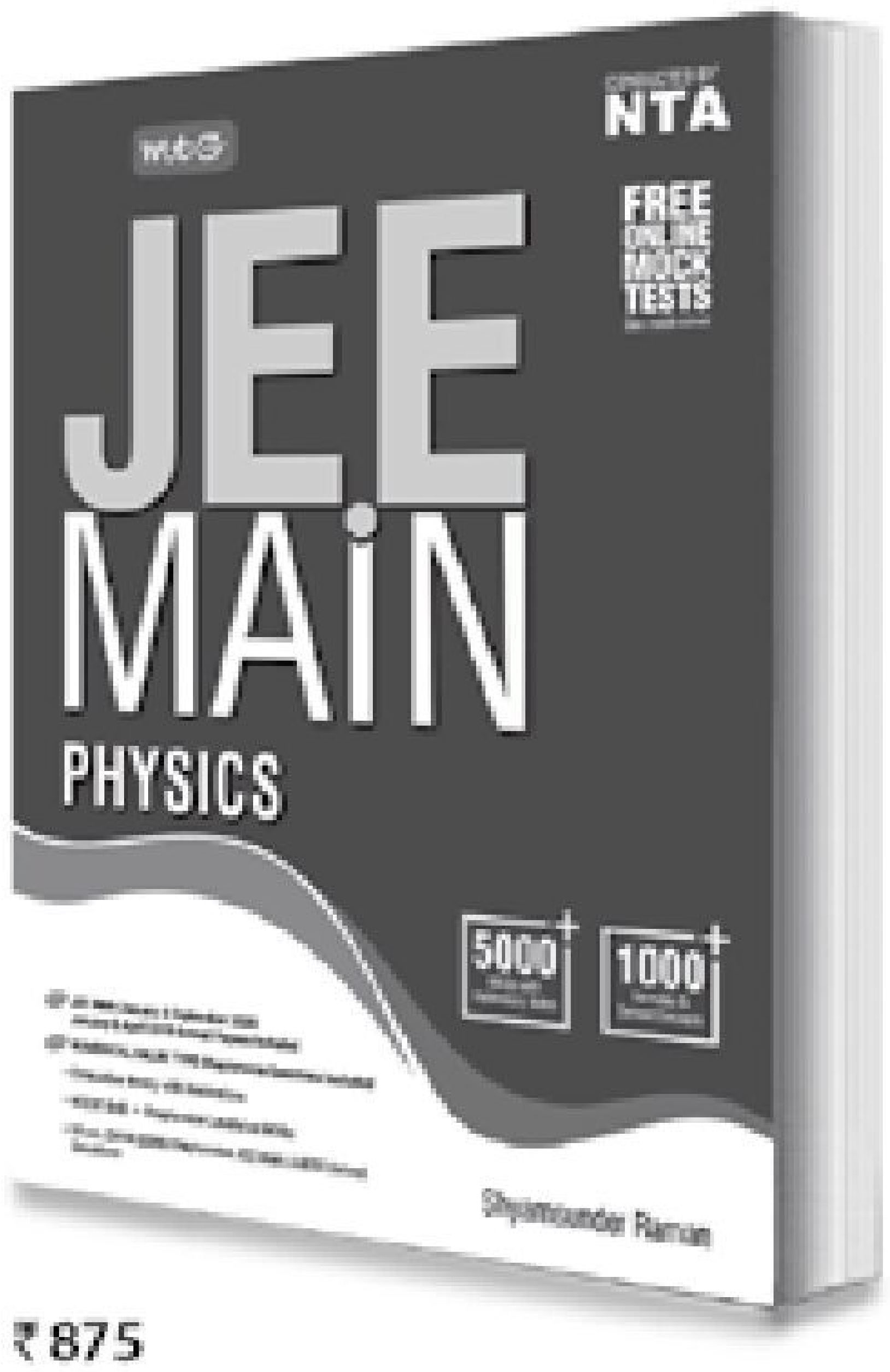
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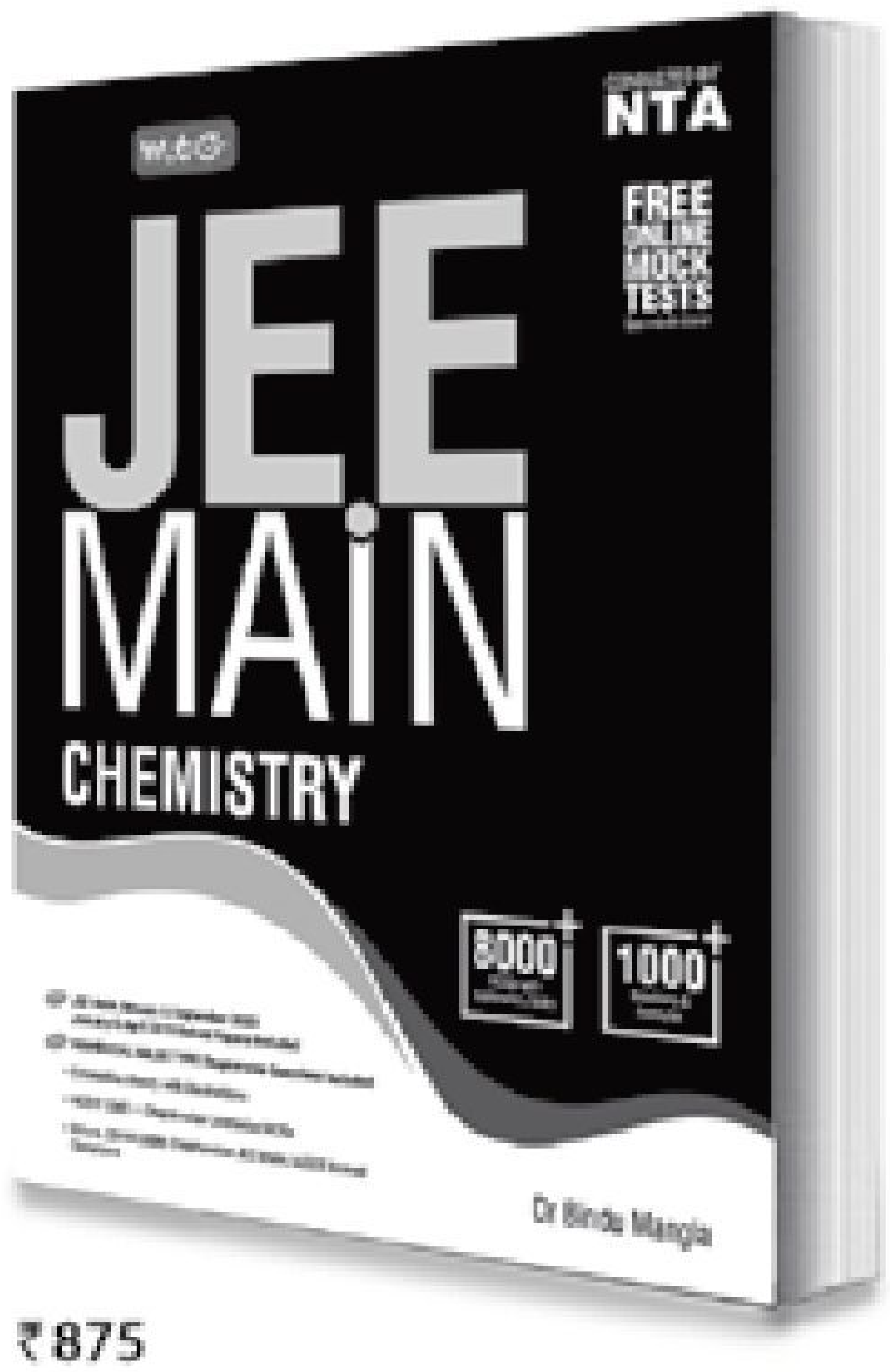
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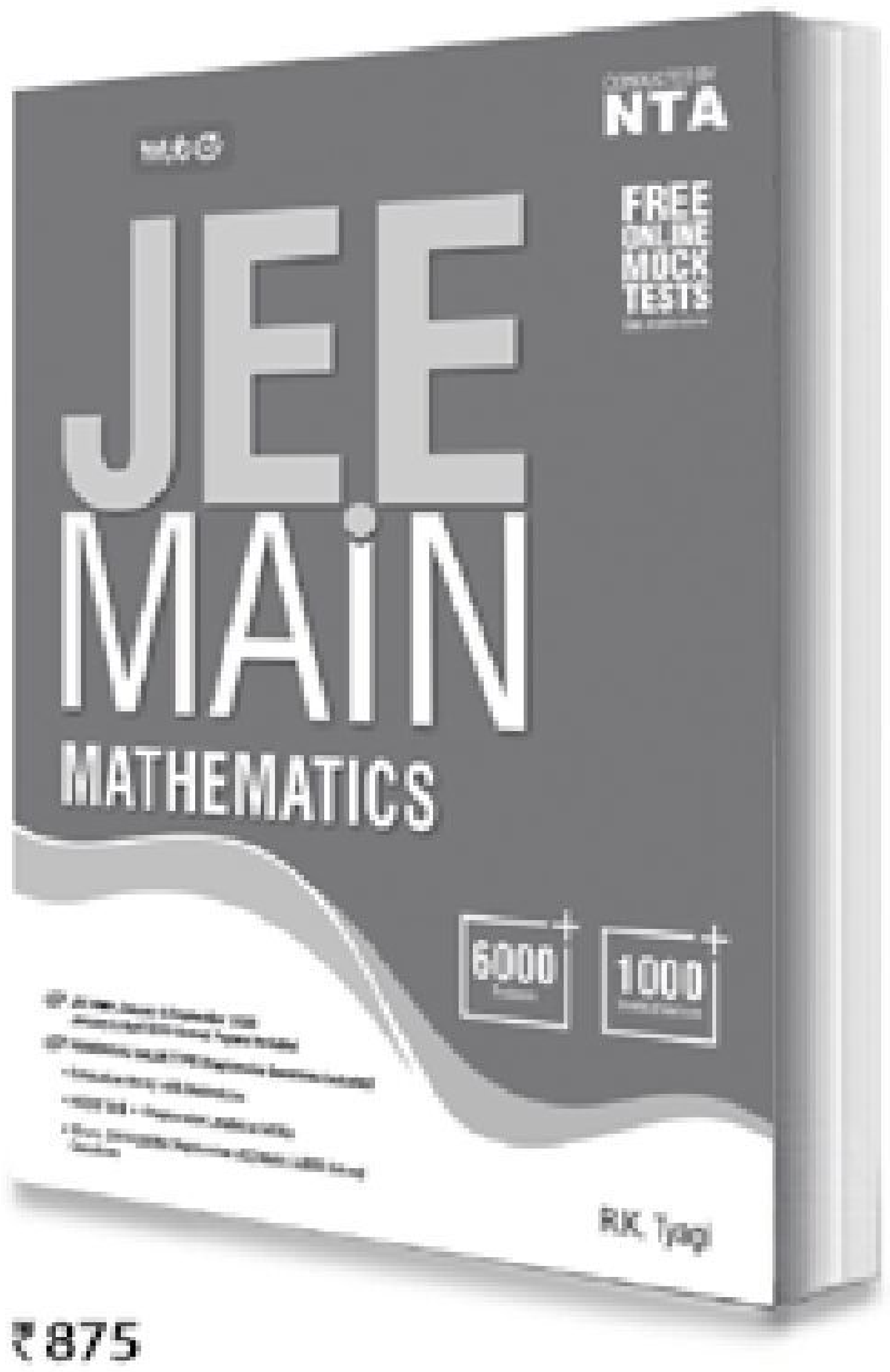
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$$\Rightarrow \quad 3\theta = n\pi + (-1)^n\left(\frac{-\pi}{6}\right),\ n \in Z$$

$$\Rightarrow \quad \theta = \frac{n\pi}{3} + (-1)^n\left(\frac{-\pi}{18}\right),\ n \in Z$$

For, $n = 1$

$$\Rightarrow \quad \theta = \frac{\pi}{3} + \frac{\pi}{18} = \frac{7\pi}{18}$$

9. (None of the options is correct) : We have,

$$\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + \ldots\ldots + \frac{2^{100}}{x^{2^{100}}+1}$$

$$= \frac{1}{1-x} + \frac{1}{1+x} + \frac{2}{1+x^2} + \frac{2^2}{1+x^4} + \ldots + \frac{2^{100}}{x^{2^{100}}+1} + \frac{1}{(x-1)}$$

$$= \frac{2}{1-x^2} + \frac{2}{1+x^2} + \frac{2^2}{1+x^4} + \ldots\ldots + \frac{2^{100}}{1+x^{2^{100}}} + \frac{1}{(x-1)}$$

$$= \frac{2^2}{1-x^4} + \frac{2^2}{1+x^4} + \ldots\ldots + \frac{2^{100}}{1+x^{2^{100}}} + \frac{1}{x-1}$$

$$= \frac{2^{101}}{1-(x^{2^{101}})} + \frac{1}{x-1}$$

Substituting $x = 2$, we get

$$\text{Required sum} = \frac{2^{101}}{1-2^{2^{101}}} + 1 = \frac{2^{101}}{1-4^{2^{100}}} + 1$$

$$= 1 - \frac{2^{101}}{4^{2^{100}}-1} = 1 - \frac{2^{101}}{(4^{2^{100}}-1)}$$

10. (a) : We have,

$(1 + x)^{20} = {}^{20}C_0 + {}^{20}C_1x + {}^{20}C_2x^2 + \ldots\ldots\ldots\ldots + {}^{20}C_{20}x^{20}$

Differentiating both sides, we get

$20(1 + x)^{19} = 1\cdot{}^{20}C_1 + 2\cdot{}^{20}C_2x + 3\cdot{}^{20}C_3x^2 + \ldots\ldots\ldots\ldots + 20\cdot{}^{20}C_{20}\,x^{19}$

Multiplying both sides by x, we get

$20x(1 + x)^{19} = 1\cdot{}^{20}C_1x + 2\cdot{}^{20}C_2x^2 + 3\cdot{}^{20}C_3x^3 + \ldots\ldots\ldots\ldots + 20\cdot{}^{20}C_{20}\,x^{20}$

Differentiating both sides, we get

$20(1 + x)^{19} + 380x(1 + x)^{18} = 1^2\cdot{}^{20}C_1 + 2^2\cdot{}^{20}C_2x + 3^2\cdot{}^{20}C_3x^2 + \ldots\ldots\ldots\ldots + 20^2\cdot{}^{20}C_{20}\,x^{19}$

Substituting $x = 1$ on both sides, we get

$20 \times 2^{19} + 380 \times 2^{18} = 1^2\cdot{}^{20}C_1 + 2^2\cdot{}^{20}C_2 + 3^2\cdot{}^{20}C_3 + \ldots\ldots\ldots\ldots + 20^2\cdot{}^{20}C_{20}$

$$\Rightarrow \quad 420 \times 2^{18} = \sum_{r=0}^{20} r^2\ {}^{20}C_r$$

11. (b) : Given that $((p \vee q) \wedge (q \to r) \wedge (\sim r)) \to (p \wedge q)$ is false.

$\Rightarrow$ $((p \vee q) \wedge (q \to r) \wedge (\sim r))$ is true and $p \wedge q$ is false.

$\Rightarrow$ $(p \vee q)$, $(q \to r)$ and $(\sim r)$ all are true.

$\Rightarrow$ r is false

Now, $(q \to r)$ is true and r is false.

$\Rightarrow$ q is false

Also, $p \vee q$ is true and q is false $\Rightarrow$ p is true.

12. (b) : Let $\cot^{-1}\sqrt{\frac{1-x}{x}} = \theta$

$$\cot\theta = \sqrt{\frac{1-x}{x}}$$

$$\therefore \quad \sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right) = \sin\theta$$

$$\text{Since, } \sin\theta = \frac{1}{\sqrt{1+\cot^2\theta}} = \frac{1}{\sqrt{1+\frac{1-x}{x}}} = \sqrt{x}$$

$$\text{Now, } \tan^{-1}\left[\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right] = \tan^{-1}\sqrt{x}$$

$$\Rightarrow \quad 2\tan^{-1}\left[\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right] = 2\tan^{-1}\sqrt{x}$$

$$= \tan^{-1}\left(\frac{2\sqrt{x}}{1-x}\right) = \phi \ \text{(say)}$$

$$\Rightarrow \quad \tan\phi = \frac{2\sqrt{x}}{1-x}$$

$$\therefore \quad \cos\left(2\tan^{-1}\left(\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)\right) = \cos\phi$$

$$\text{and } \cos\phi = \frac{1}{\sqrt{1+\tan^2\phi}}$$

$$= \frac{1}{\sqrt{1+\frac{4x}{(1-x)^2}}} = \frac{(1-x)}{(1+x)} = f(x) \ \text{(say)}$$

$$\Rightarrow \quad f'(x) = \frac{-1\times(1+x)-(1-x)}{(1+x)^2} = \frac{-2}{(1+x)^2}$$

$$\Rightarrow \quad (1-x)^2 f'(x) = \frac{-2(1-x)^2}{(1+x)^2} = -2[f(x)]^2$$

$$\Rightarrow \quad (1-x)^2 f'(x) + 2[f(x)]^2 = 0$$

13. (b) : We have, $A=\begin{bmatrix}\frac{1}{\sqrt5} & \frac{2}{\sqrt5}\\ \frac{-2}{\sqrt5} & \frac{1}{\sqrt5}\end{bmatrix} \Rightarrow A^T=\begin{bmatrix}\frac{1}{\sqrt5} & \frac{-2}{\sqrt5}\\ \frac{2}{\sqrt5} & \frac{1}{\sqrt5}\end{bmatrix}$

$$\therefore\quad AA^T=\begin{bmatrix}\frac{1}{\sqrt5} & \frac{2}{\sqrt5}\\ \frac{-2}{\sqrt5} & \frac{1}{\sqrt5}\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt5} & \frac{-2}{\sqrt5}\\ \frac{2}{\sqrt5} & \frac{1}{\sqrt5}\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

Now, as $Q = A^TBA$

$\therefore\quad Q^2 = (A^TBA)(A^TBA) = A^TB^2A$

$Q^3 = A^TB^3A$

....

....

$Q^{2021} = A^TB^{2021}A$

Let $P = AQ^{2021}A^T$, then

$P = A(A^TB^{2021}A)A^T = (AA^T)B^{2021}(AA^T) = B^{2021}$

$$B=\begin{bmatrix}1 & 0\\ i & 1\end{bmatrix},\ B^2=\begin{bmatrix}1 & 0\\ i & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ i & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 2i & 1\end{bmatrix}$$

$$B^3=\begin{bmatrix}1 & 0\\ 2i & 1\end{bmatrix}\begin{bmatrix}1 & 0\\ i & 1\end{bmatrix}=\begin{bmatrix}1 & 0\\ 3i & 1\end{bmatrix}$$

....

....

$$B^{2021}=\begin{bmatrix}1 & 0\\ 2021i & 1\end{bmatrix}$$

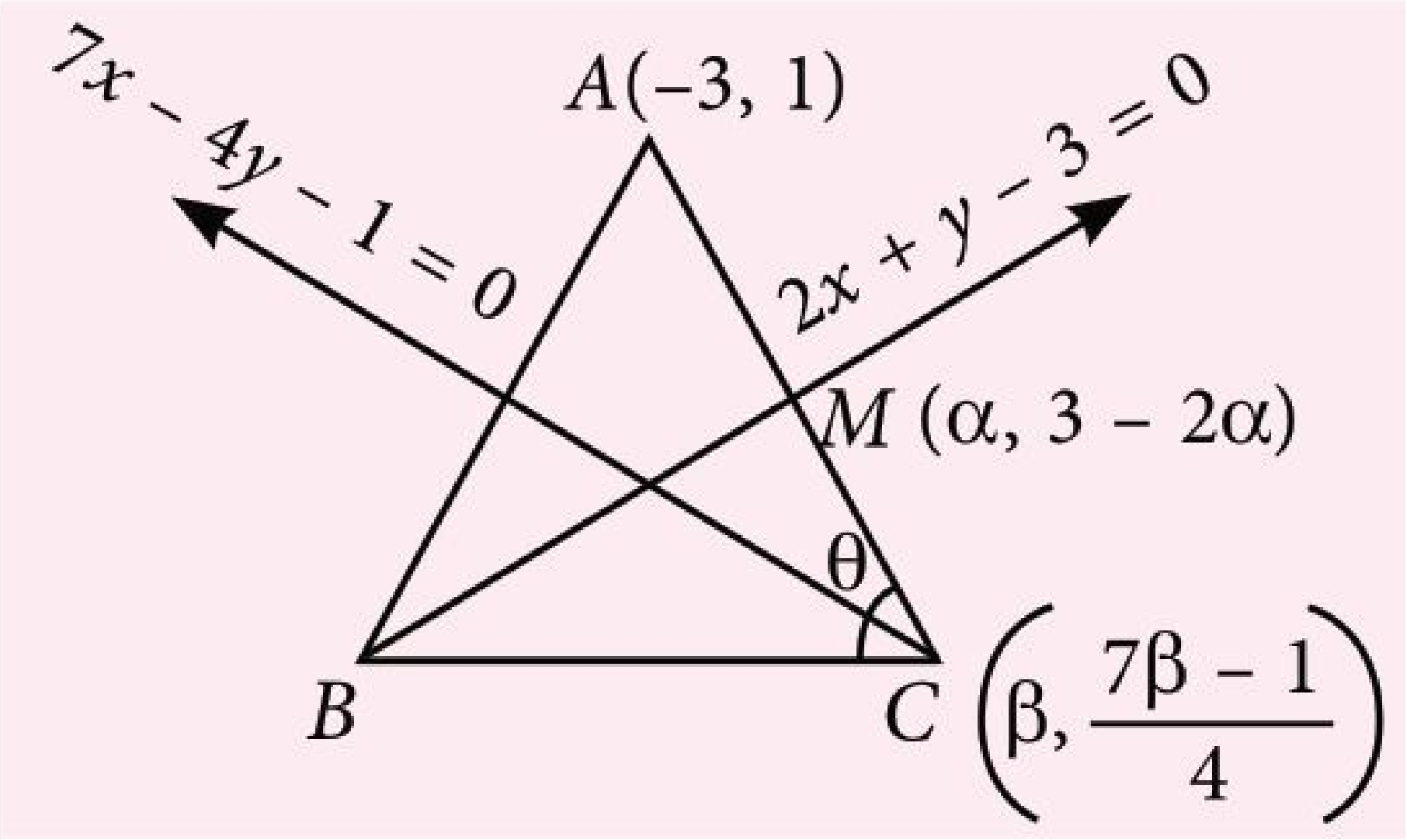
$$\therefore\ P^{-1}=(B^{2021})^{-1}=\begin{bmatrix}1 & 0\\ -2021i & 1\end{bmatrix}$$

14. (b) : Let M be the midpoint AC.

Let coordinates of $M \equiv (\alpha, 3-2\alpha)$ and $C\equiv\left(\beta, \frac{7\beta-1}{4}\right)$.

Now, $\frac{\beta-3}{2}=\alpha \Rightarrow 2\alpha=\beta-3$ ...(i)

and $\frac{7\beta-1}{4}+1=6-4\alpha$



$\Rightarrow\quad 7\beta + 3 = 24 - 16\alpha$

$\Rightarrow\quad 7\beta + 3 = 24 - 8 \times (\beta - 3)$ [Using (i)]

$\Rightarrow\quad 15\beta = 48 - 3$

$\Rightarrow\quad 15\beta = 45$

$\Rightarrow\quad \beta = 3$ and $\frac{7\beta-1}{4}=5$

$\therefore\quad$ Coordinates of $C \equiv (3, 5)$

Now, slope of $AC=\frac{5-1}{3+3}=\frac{2}{3}$

Slope of angle bisector $=\frac{7}{4}$

$$\text{Now, }\tan\frac{\theta}{2}=\left|\frac{\frac{7}{4}-\frac{2}{3}}{1+\frac{14}{12}}\right|=\frac{1}{2}$$

$$\therefore\ \tan\theta=\left|\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right|=\left|\frac{2\times\frac{1}{2}}{1-\frac{1}{4}}\right|=\frac{4}{3}$$

15. (a) : We have, mean $(\bar{x})=\frac{\Sigma x_i}{20}=10 \Rightarrow \Sigma x_i=200$

$\therefore\quad$ Correct $\Sigma x_i = 200 - 25 + 35 = 210$

$\therefore\quad$ Correct mean $(\bar{x})=\frac{210}{20}=10.5=\alpha$

Also, S.D $(\sigma) = 2.5$

$$\Rightarrow\ \sqrt{\frac{\Sigma x_i^2}{20}-10^2}=2.5$$

$\Rightarrow\quad \Sigma x_i^2 = 106.25 \times 20 = 2125$

Correct $\Sigma x_i^2 = 2125 - 625 + 1225 = 2725$

Correct S.D. $(\sqrt{\beta})=\sqrt{\frac{2725}{20}-10.5^2}$

$=\sqrt{136.25-110.25}=\sqrt{26}=\sqrt{\beta} \Rightarrow \beta=26$

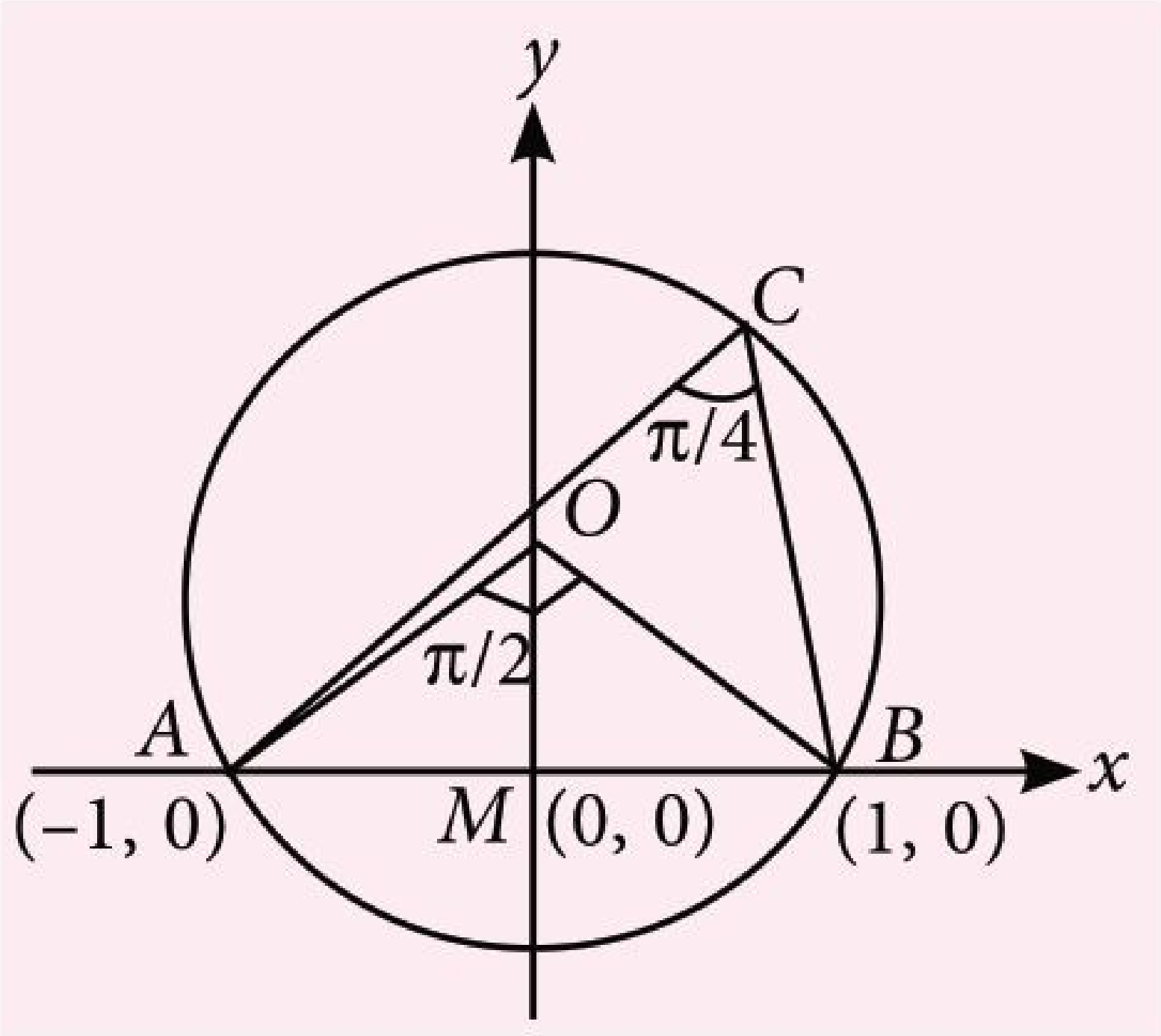
$\therefore\quad (\alpha, \beta)$ is $(10.5, 26)$

16. (d) : $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ represents a circle passing through $(1, 0)$ and $(-1, 0)$ and subtending an angle of $\frac{\pi}{4}$ at major arc of the circle.

$\therefore\quad$ Angle subtending at centre O is $\frac{\pi}{2}$.

Since, O lies on y-axis (from symmetry), therefore

$\angle MOB=\angle MOA=\frac{\pi}{4}$



$\Rightarrow\ \angle OBM = \frac{\pi}{4}$

$\Rightarrow\ OM = MB$

$\therefore\ OM = 1\ [\because MB = 1]$

Thus, coordinates of O are $(0, 1)$ and

$OB = \text{Radius} = \sqrt{1^2 + 1^2}$

$= \sqrt{2}$

$\therefore\ \arg\left(\frac{z-1}{z+1}\right)$ represents a circle with centre at $(0, 1)$ and radius $\sqrt{2}$.

17. (c) : We have, $y^2 = 30x = 4 \times \frac{15}{2}x$

$\Rightarrow\ a = \frac{15}{2}$

Tangent to this parabola is $y = mx + \frac{15}{2m}$

This line passes through $(-30, 0)$.

$\therefore\ 0 = -30m + \frac{15}{2m} \Rightarrow m^2 = \frac{1}{4}$

$\Rightarrow\ m = \pm\frac{1}{2}$

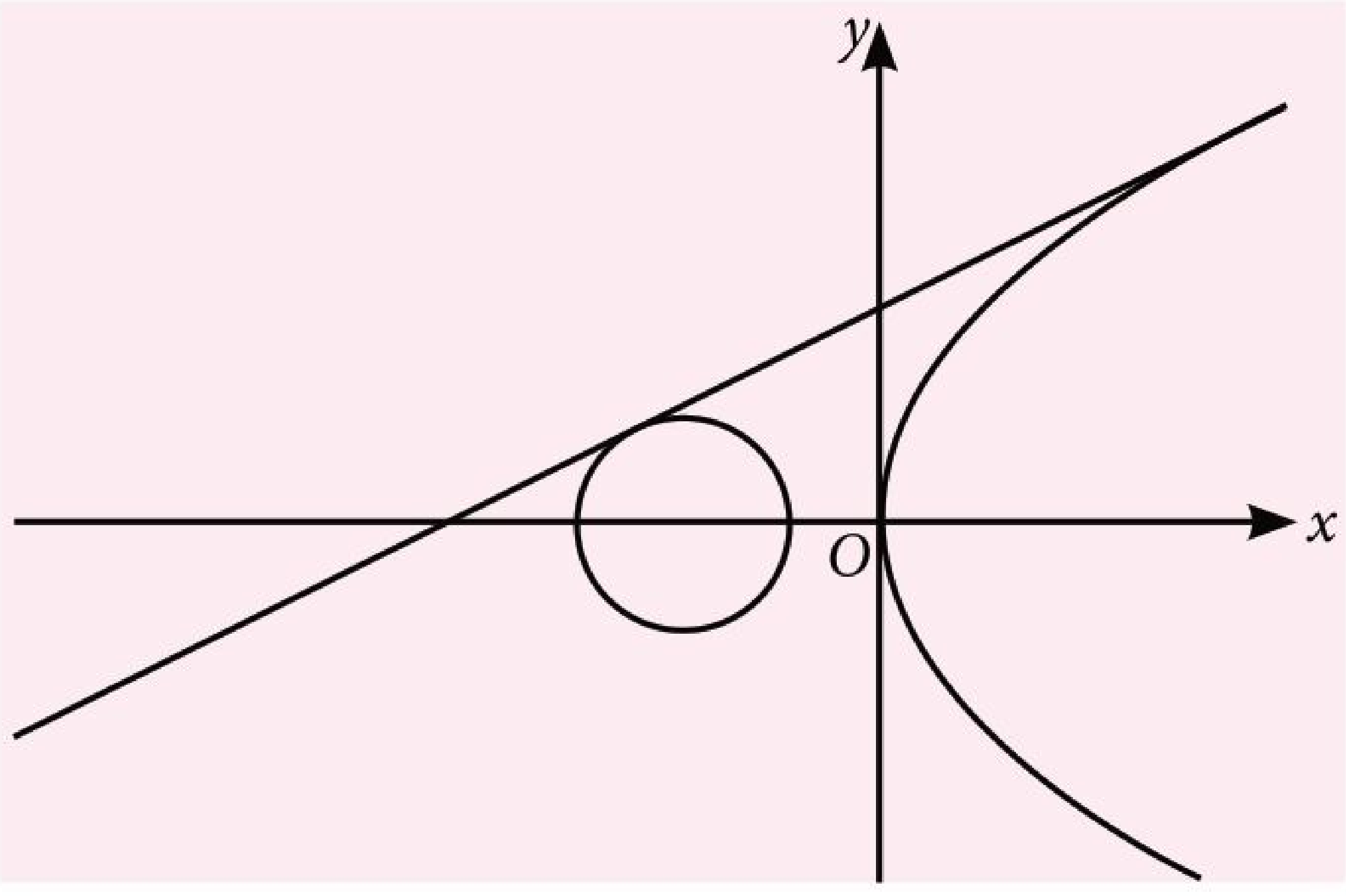
$\therefore$ Equation of the tangent is $y = \frac{x}{2} + 15$

or $y = \frac{-x}{2} - 15$,

i.e., $y - \frac{x}{2} - 15 = 0$ or $y + \frac{x}{2} + 15 = 0$

The given circle is $x^2 + y^2 + 30x + \frac{675}{4} = 0$

or $(x+15)^2 + y^2 = \frac{225}{4}$



$\therefore$ Centre is $(-15, 0)$ and radius r is $\frac{15}{2}$.

Length of perpendicular from centre to the chord

$$= \left|\frac{0 + \frac{15}{2} - 15}{\sqrt{5}/2}\right| = 3\sqrt{5} \quad\text{or}\quad \left|\frac{0 - \frac{15}{2} + 15}{\sqrt{5}/2}\right| = 3\sqrt{5}$$

Length of the chord $= 2\sqrt{R^2 - (3\sqrt{5})^2}$

$$= 2 \times \sqrt{\frac{225}{4} - 45} = 2 \times \sqrt{\frac{45}{4}} = 3\sqrt{5}$$

18. (c) : Let $f(x) = \sqrt{\left(\frac{x+1}{x-1}\right)^2 + \left(\frac{x-1}{x+1}\right)^2 - 2}$

$$= \left|\frac{x+1}{x-1} - \frac{x-1}{x+1}\right|$$

$$\Rightarrow\ f(x) = 4 \times \left|\frac{x}{x^2-1}\right|$$

$\because$ $f(-x) = f(x)$, *i.e.*, $f(x)$ is an even function.

$$\therefore\ I = 2 \times \int_0^{\frac{1}{\sqrt{2}}} 4\left|\frac{x}{x^2-1}\right| dx = 8 \times \int_0^{\frac{1}{\sqrt{2}}} \frac{x}{1-x^2}\,dx$$

$$= -4\int_0^{\frac{1}{\sqrt{2}}} \frac{-2x\,dx}{1-x^2} = -4\left[\log_e(1-x^2)\right]_0^{\frac{1}{\sqrt{2}}}$$

$$= -4 \times \log_e\left(\frac{1}{2}\right) = 4\log_e 2 = \log_e 16$$

19. (c) : Given, $a + ar + ar^2 + \ldots\ldots\ldots\ \infty = 15$

$$\Rightarrow\ \frac{a}{1-r} = 15 \Rightarrow a = 15(1-r) \qquad \ldots\text{(i)}$$

Also, $a^2 + a^2r^2 + a^2r^4 + \ldots\ldots\ \infty = 150$

$$\Rightarrow\ \frac{a^2}{1-r^2} = 150 \Rightarrow \left(\frac{a}{1-r}\right) \times \frac{a}{(1+r)} = 150$$

$$\Rightarrow\ \frac{a}{1+r} = 10 \Rightarrow a = 10(1+r) \qquad \ldots\text{(ii)}$$

From (i) and (ii), we get

$15(1 - r) = 10(1 + r)$

$\Rightarrow\ 3 - 3r = 2 + 2r \Rightarrow r = \frac{1}{5}$

From (i), $a = 15\left(1 - \frac{1}{5}\right) = 15 \times \frac{4}{5} = 12$

Now, $ar^2 + ar^4 + ar^6 + \ldots\ldots\ \infty = \frac{ar^2}{1-r^2}$

$$= \frac{12 \times \frac{1}{25}}{1 - \frac{1}{25}} = \frac{12}{24} = \frac{1}{2}$$

20. (c) : We have, $(y + 1)\tan^2 x\,dx + \tan x\,dy + y\,dx = 0$

$\Rightarrow\ (y + 1)(\sec^2 x - 1)dx + \tan x\,dy + y\,dx = 0$

$\Rightarrow \quad (y\sec^2 x\,dx + \tan x\,dy) + \sec^2 x\,dx - dx = 0$

$\Rightarrow \quad d(y\cdot\tan x) + d(\tan x) - dx = 0$

Integrating both sides, we get

$y\tan x + \tan x - x = C \Rightarrow (y+1)\tan x - x = C$

$$\Rightarrow \quad y = \frac{C+x}{\tan x} - 1$$

$$\text{Now, } \lim_{x\to 0^+} xy(x) = \lim_{x\to 0^+}\left[\frac{x(C+x)}{\tan x} - x\right]$$

$$= \lim_{x\to 0^+}\left[\left(\frac{x}{\tan x}\right)(C+x) - x\right]$$

$$= \lim_{x\to 0^+}[(C+x)-x] \quad \left[\because \lim_{x\to 0^+}\left(\frac{x}{\tan x}\right) = 1\right]$$

$= C = 1$ (given)

$$\therefore \quad y(x) = \frac{x+1}{\tan x} - 1$$

$$\Rightarrow \quad y\left(\frac{\pi}{4}\right) = \left(\frac{\frac{\pi}{4}+1}{\tan\left(\frac{\pi}{4}\right)}\right) - 1 = \frac{\pi}{4}$$

Numerical value Type

1. (136 or 680) : Let $t_n = n\cdot{}^nP_n$

$$= n \times \frac{n!}{(n-n)!} = n\cdot n! = (n+1-1)n!$$

$= (n+1)! - n!$

$t_1 = 2! - 1!$

$t_2 = 3! - 2!$

$t_3 = 4! - 3!$ ...., $t_{15} = 16! - 15!$

$$\therefore \quad \sum_{n=1}^{15} t_n = 16! - 1 = {}^qP_r - s$$

$\Rightarrow \quad {}^qP_r = 16! - 1 + s$

Since, ${}^qP_r$ is an integer, s can take values 1 or 0.

$s = 0$ is not possible as there is no such q and r for which ${}^qP_r = 16! - 1$

$s = 1 \Rightarrow {}^qP_r = 16! \Rightarrow q = r = 16$ or $q = 16$ and $r = 15$

$\therefore \quad {}^{q+s}C_{r-s} = {}^{17}C_{15}$ or ${}^{17}C_{14}$, *i.e.*, 136 or 680

2. (26) : Let any point on the given line be $P(2r+1, r+3, 2r+4)$

Foot of perpendicular from point P on the plane is given by

$$\frac{x-(2r+1)}{1} = \frac{y-(r+3)}{-2} = \frac{z-(2r+4)}{-1}$$

$$= \frac{-[(2r+1)-2(r+3)-(2r+4)-3]}{(1)^2+(-2)^2+(-1)^2} = \frac{r+6}{3}$$

[$\because$ The foot of perpendicular (x, y, z) of a point (x_1, y_1, z_1) in a plane $ax + by + cz + d = 0$ is given by

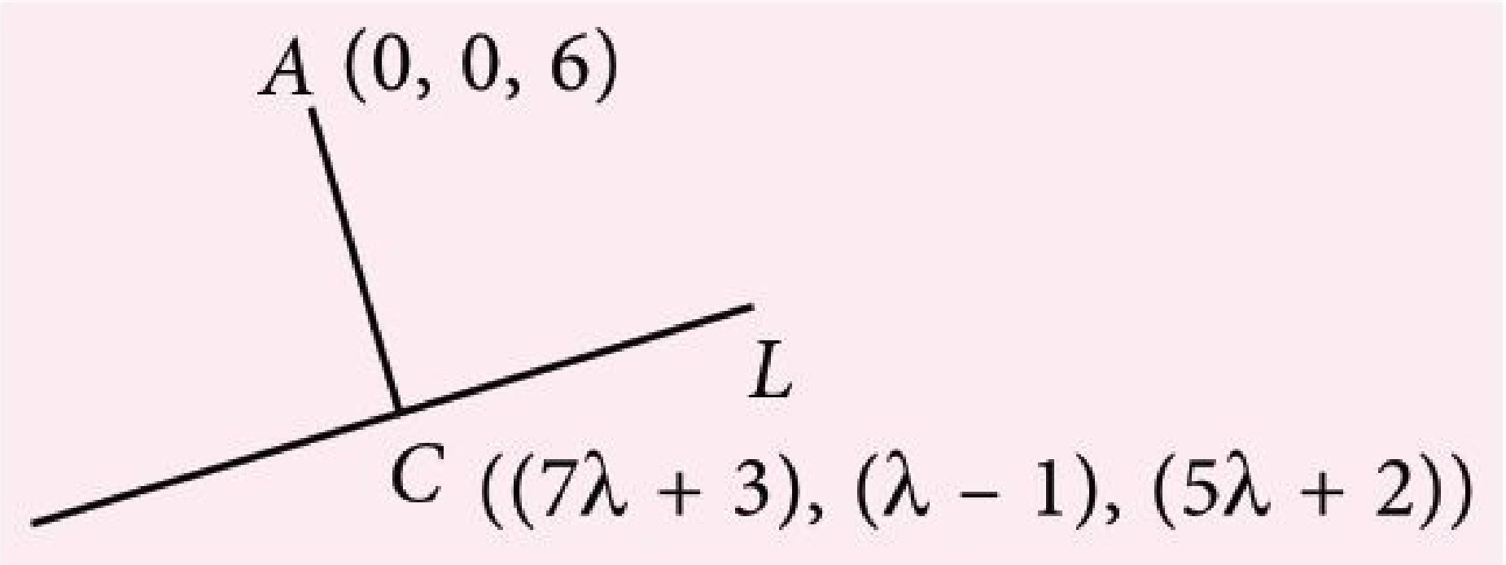
$$\frac{x-x_1}{a} = \frac{y-y_1}{b} = \frac{z-z_1}{c} = \frac{-(ax_1+by_1+cz_1+d)}{a^2+b^2+c^2}]$$

$$\Rightarrow \quad x = \frac{7r+9}{3},\ y = \frac{r-3}{3},\ z = \frac{5r+6}{3}$$

Projection line L is

$$\frac{x-3}{7} = \frac{y+1}{1} = \frac{z-2}{5} \qquad \text{...(i)}$$

Direction ratios of AC are $(7\lambda + 3, \lambda - 1, 5\lambda - 4)$



Now, as AC is perpendicular to the projection line (i).

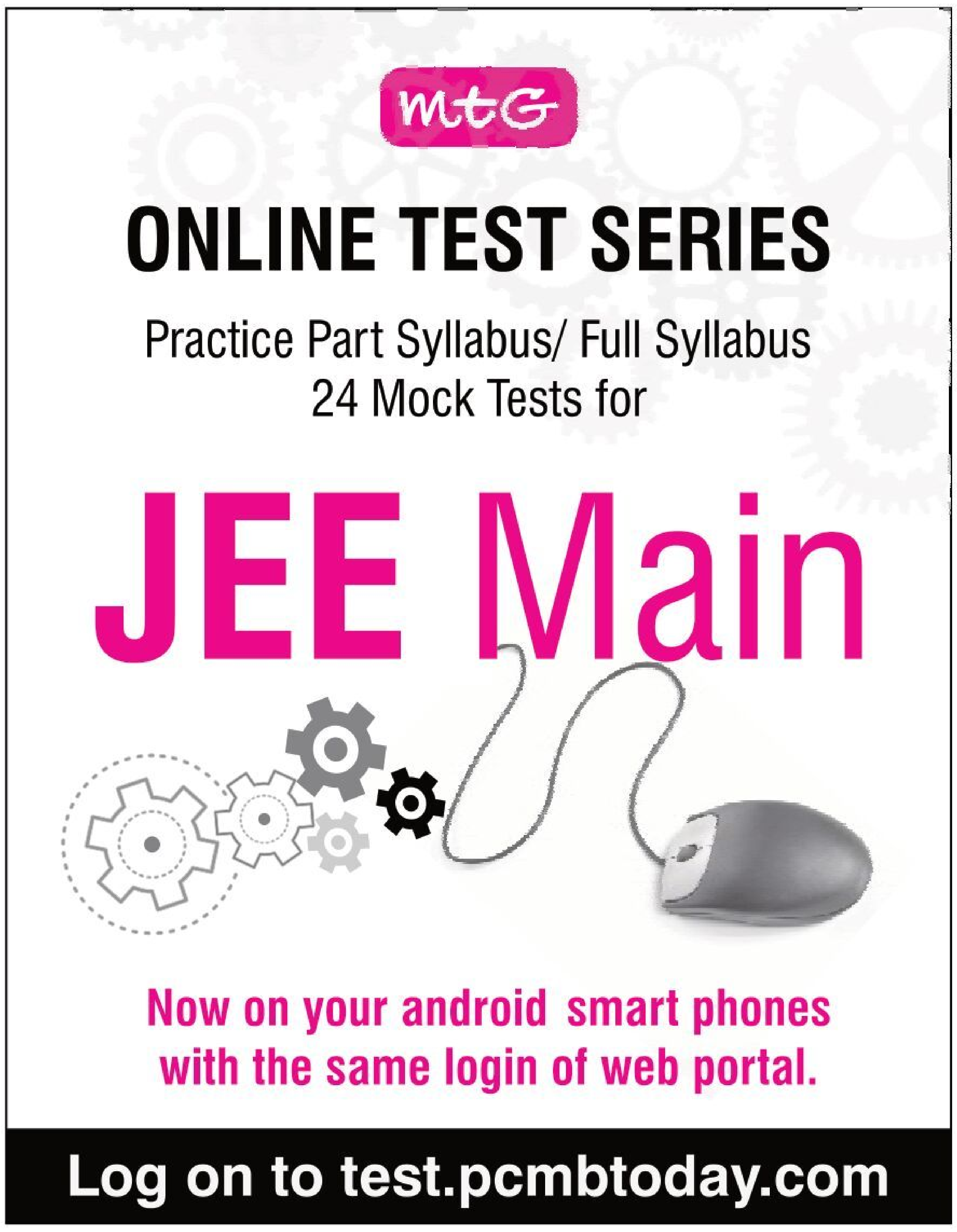
$\therefore \quad 7(7\lambda+3) + 1(\lambda-1) + 5(5\lambda-4) = 0$

$\Rightarrow \quad 49\lambda + \lambda + 25\lambda + 21 - 1 - 20 = 0$

$\Rightarrow \quad \lambda = 0 \Rightarrow C(3, -1, 2)$

Now, $d^2 = (AC)^2 = (3-0)^2 + (-1-0)^2 + (2-6)^2$

$= 9 + 1 + 16 = 26$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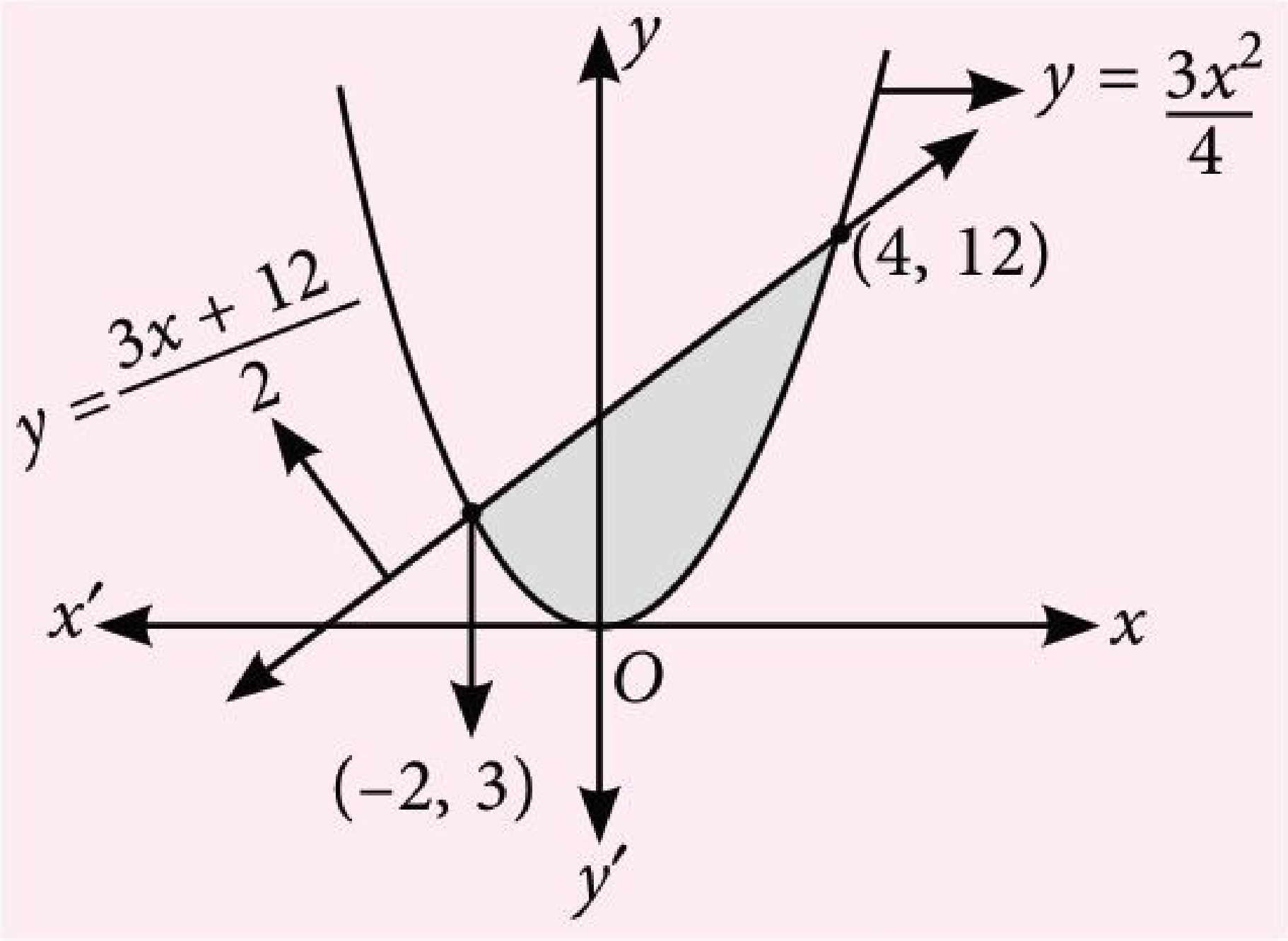
3. (27) : We have, $\frac{3x^2}{4} \le y \le \frac{3x+12}{2}$

For point of intersection, we have $3x^2 = 6x + 24$

$\Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow x = 4$ and $x = -2$



$\therefore$ Required area $= \int_{-2}^{4} \left(\frac{3x+12}{2} - \frac{3x^2}{4}\right) dx$

$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4}\right]_{-2}^{4}$

$= (12 + 24 - 16) - (3 - 12 + 2) = 20 - (-7) = 27$

4. (16) : Let $P(x, y)$ be any point, according to the question, we have

$$(x - 0)^2 + (y - 0)^2 + (x - 1)^2 + (y - 0)^2 + (x - 0)^2 + (y - 1)^2 + (x - 1)^2 + (y - 1)^2 = 18$$

$\Rightarrow 4x^2 + 4y^2 - 4x - 4y + 4 = 18$

$\Rightarrow x^2 + y^2 - x - y = \frac{7}{2}$

$\Rightarrow \left(x - \frac{1}{2}\right)^2 + \left(y - \frac{1}{2}\right)^2 = 4$, which is equation of circle with radius, $r = 2 \Rightarrow$ Diameter, $d = 4$

$\therefore d^2 = 16$

5. (66) : Given, $\frac{2}{x-1} - \frac{1}{x-2} = \frac{2}{k}$

$\Rightarrow \frac{2x - 4 - x + 1}{x^2 - 3x + 2} = \frac{2}{k}$

$\Rightarrow k(x - 3) = 2x^2 - 6x + 4$

$\Rightarrow 2x^2 - (6 + k)x + 4 + 3k = 0$

This equation has no real roots.

$\Rightarrow D < 0$

$\Rightarrow (k + 6)^2 - 8(4 + 3k) < 0$

$\Rightarrow k^2 + 12k + 36 - 32 - 24k < 0$

$\Rightarrow k^2 - 12k + 4 < 0$

$\Rightarrow (k - 6)^2 < 32$

$\Rightarrow -4\sqrt{2} < k - 6 < 4\sqrt{2}$

$\Rightarrow 6 - 4\sqrt{2} < k < 6 + 4\sqrt{2}$

$\Rightarrow 0.3431 < k < 11.66$

$\therefore$ k can take values 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 and 11

$\therefore$ Sum of values of $k = \frac{11 \times 12}{2} = 66$

6. (14) : $\lim_{x \to 0^-} f(x) = a \sin\left[\frac{\pi}{2}(0 - 1)\right] = -a$ ...(i)

Also, $f(0) = -a$ ...(ii)

Since, $\tan(2x) = (2x) + \frac{1}{3}(2x)^3 + \frac{2}{15}(2x)^5 + ...$

and $\sin(2x) = (2x) - \frac{1}{6}(2x)^3 + \frac{(2x)^5}{5!} - ....$

$\therefore \tan(2x) - \sin(2x) = 4x^3 + \frac{2}{15}(2x)^5 - \frac{(2x)^5}{5!} + ...$

$$\lim_{x \to 0} \frac{\tan(2x) - \sin(2x)}{bx^3} = \lim_{x \to 0} \frac{4x^3 + \frac{2}{15}(2x)^5 - \frac{(2x)^5}{5!}.....}{bx^3}$$

$= \frac{4}{b}$ ...(iii)

Since, $f(x)$ is continuous at $x = 0$.

$\therefore \frac{4}{b} = -a \Rightarrow ab = -4$ (Using (i), (ii) and (iii))

So, $10 - ab = 14$

7. (36) : Let the circumference of circle be x m.

$\Rightarrow 2\pi r = x \Rightarrow r = \frac{x}{2\pi}$m

Also, perimeter of square = $(36 - x)$m

$\Rightarrow 4a = 36 - x \Rightarrow a = \left(\frac{36 - x}{4}\right)$m

Now, area of circle $\pi r^2 = \pi \times \left(\frac{x}{2\pi}\right)^2 = \frac{x^2}{4\pi}$

and area of square, $a^2 = \left(\frac{36 - x}{4}\right)^2 = 81 + \frac{x^2}{16} - \frac{9}{2}x$

$\therefore$ Sum of areas, $A = \frac{x^2}{4\pi} + \frac{x^2}{16} + 81 - \frac{9}{2}x$

For minimum area, put $\frac{dA}{dx} = 0$

$\Rightarrow \frac{dA}{dx} = \frac{x}{2\pi} + \frac{x}{8} - \frac{9}{2} = 0$

$\Rightarrow (4 + \pi)x = 36\pi$

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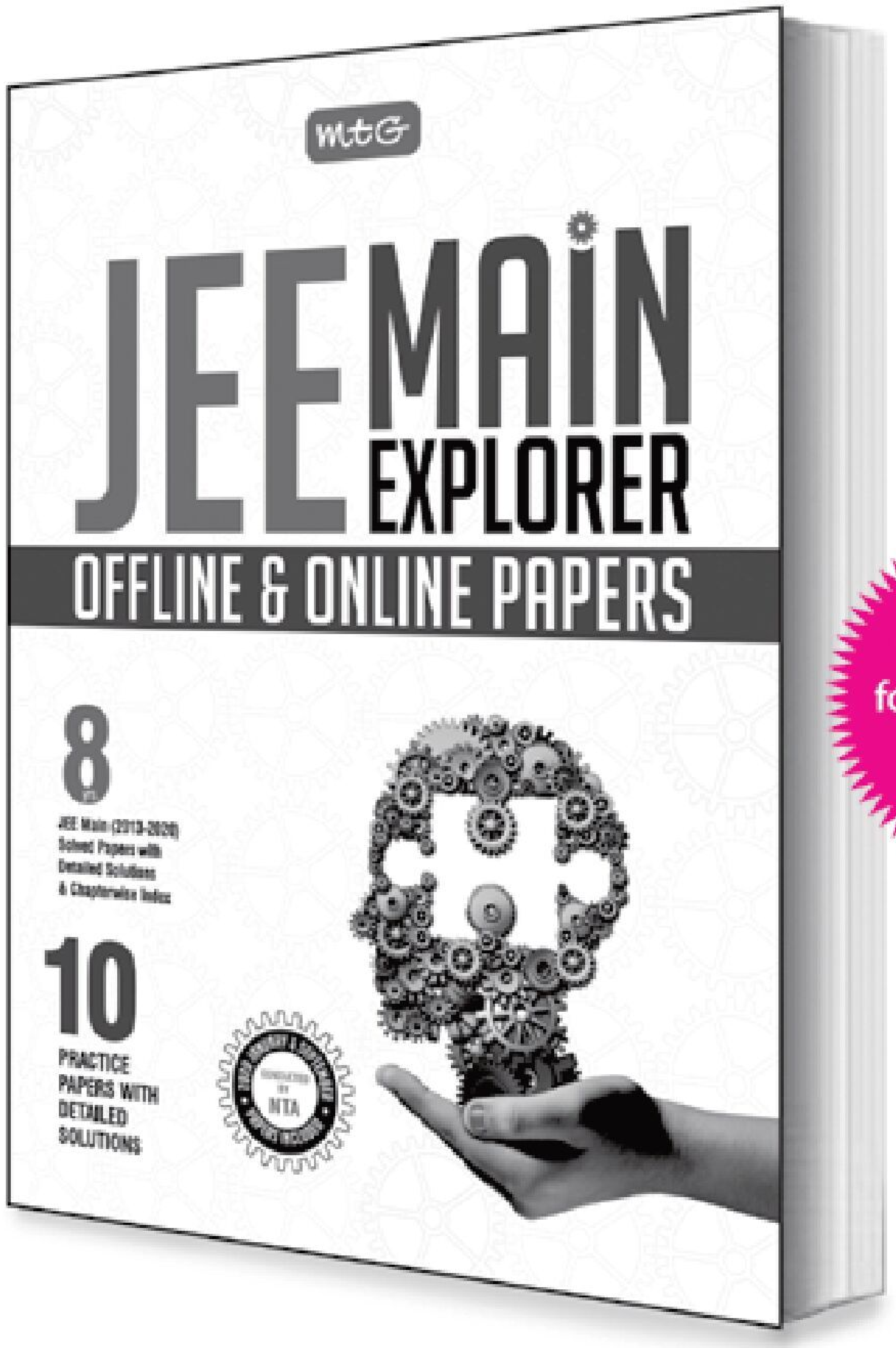
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$$\Rightarrow \quad x=\frac{36\pi}{\pi+4}$$

Now, as $\frac{d^2A}{dx^2}=\frac{1}{2\pi}+\frac{1}{8}>0\ \forall x$

$\therefore$ A is minimum when $x=\frac{36\pi}{\pi+4}$.

Now, k = Circumference of circle $=x=\left(\frac{36\pi}{\pi+4}\right)$

$$\therefore \quad \left(\frac{\pi+4}{\pi}\right)\times k=\left(\frac{\pi+4}{\pi}\right)\times\frac{36\pi}{\pi+4}=36$$

8. (13) : Clearly,

$$z=\frac{1-i\sqrt{3}}{2}=\frac{-1-i\sqrt{3}}{2}+1=1+\omega^2=-\omega$$

$$[\because\ 1+\omega+\omega^2=0]$$

Now, $z+\frac{1}{z}=-\omega-\frac{1}{\omega}=1,\ z^2+\frac{1}{z^2}=\omega^2+\frac{1}{\omega^2}=-1,$

$$z^3+\frac{1}{z^3}=-\omega^3-\frac{1}{\omega^3}=-2,$$

$$z^4+\frac{1}{z^4}=\omega^4+\frac{1}{\omega^4}=\omega+\frac{1}{\omega}=-1,$$

$$z^5+\frac{1}{z^5}=-\omega^5-\frac{1}{\omega^5}=-\left(\omega^2+\frac{1}{\omega^2}\right)=1,$$

$$z^6+\frac{1}{z^6}=\omega^6+\frac{1}{\omega^6}=2$$

$z^7+\frac{1}{z^7}=-\omega^7-\frac{1}{\omega^7}=-\omega-\frac{1}{\omega}=1$ and so on.

$$\text{Now, } 21+\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3+\ldots\ldots$$

$$\ldots\ldots+\left(z^{21}+\frac{1}{z^{21}}\right)^3$$

= 21 + (1 − 1 − 8 − 1 + 1 + 8) × 3 + 1 − 1 − 8
= 21 + [0 − 8] = 13

9. (52) : The units place must be either 0, 4 or 6 for the number to be even.

☐ ☐ ☐
4 5 3 ← No. of ways

Number of 3 digit numbers which are even
= 4 × 5 × 3 = 60
But this includes even numbers with 0 at hundreds place.
Number of such numbers = 2 × 4 = 8

$\therefore$ Required number of 3 digit even numbers
= 60 − 8 = 52

10. (40) : We have, $\log_e(x+y)=4xy$

Substituting $x=0$, we get

$\log_e y=0 \Rightarrow y=1$

Thus, $x=0 \Rightarrow y=1$

$$\text{Clearly, } \frac{1}{(x+y)}\left(1+\frac{dy}{dx}\right)=4\times\left(y+\frac{x\,dy}{dx}\right) \qquad \ldots\text{(i)}$$

At $x=0$, $y=1$, we have

$$1+\frac{dy}{dx}=4\times\left(1+0\cdot\frac{dy}{dx}\right) \Rightarrow \frac{dy}{dx}=3$$

Also, from (i), we have $1+\frac{dy}{dx}=4(x+y)\left(y+\frac{x\,dy}{dx}\right)$

$$\Rightarrow \quad \frac{d^2y}{dx^2}=4\left(1+\frac{dy}{dx}\right)\left(y+\frac{x\,dy}{dx}\right)$$

$$+4(x+y)\left(\frac{dy}{dx}+\frac{dy}{dx}+\frac{xd^2y}{dx^2}\right)$$

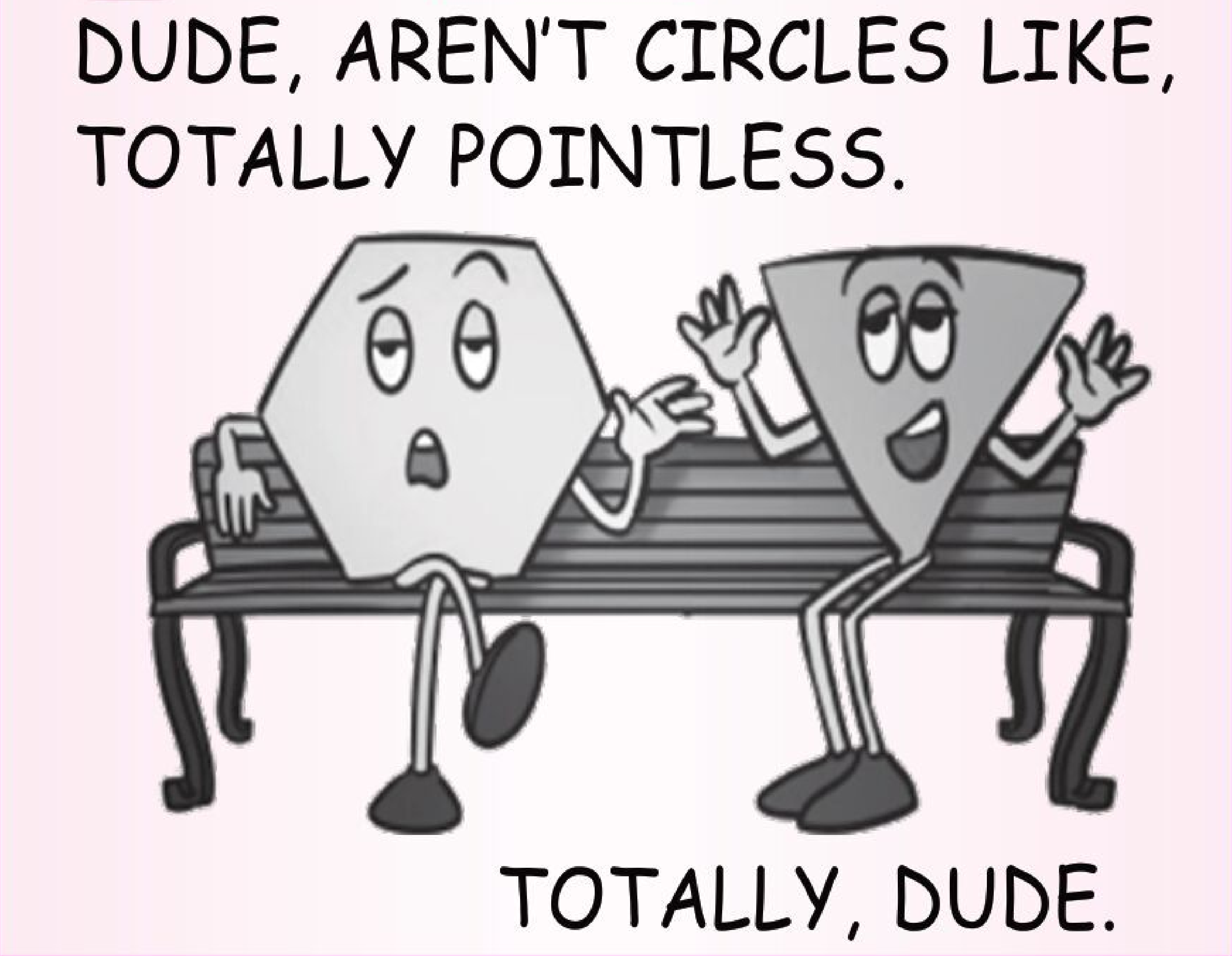
Substituting $x=0$, $y=1$ and $\frac{dy}{dx}=3$, we get

$$\frac{d^2y}{dx^2}=4(1+3)\left(1+0\cdot\frac{dy}{dx}\right)+4(0+1)\left(2\times3+0\cdot\frac{d^2y}{dx^2}\right)$$

= 4 × 4 + 4 × 6 = 16 + 24 = 40

* * *

COMIC CAPSULE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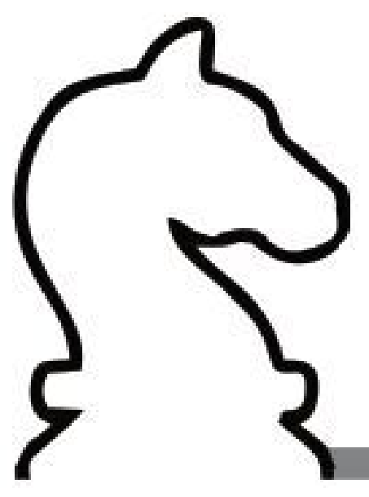
TOTALLY, DUDE.

Challenging PROBLEMS ON JEE



Single Option Correct Type

1. The sequence $< a_n >$ is defined by $a_1 = 0$ and $a_{n+1} = a_n + 4n + 3, n \geq 1$. Then the value of

$$\lim_{n\to\infty} \frac{\sqrt{a_n}+\sqrt{a_{4n}}+\sqrt{a_{4^2 n}}+....+\sqrt{a_{4^{10} n}}}{\sqrt{a_n}+\sqrt{a_{2n}}+\sqrt{a_{2^2 n}}+...+\sqrt{a_{2^{10} n}}}$$ is

(a) 600 (b) 683 (c) 800 (d) 863

2. The solution of the system of equations $x + y = \frac{\pi}{2}$, $\sin x + \sin y = \sqrt{2}$ is $\{x, y\}$, then

(a) $x = 2k\pi + \frac{\pi}{4}, y = \frac{\pi}{4} - 2k\pi, k \in I$

(b) $x = 2k\pi - \frac{\pi}{2}, y = \frac{\pi}{2} + k\pi, k \in I$

(c) $x = 2k\pi + \frac{\pi}{3}, y = \frac{\pi}{3} - 2k\pi, k \in I$

(d) $x = k\pi + \frac{\pi}{4}, y = \frac{\pi}{4} - k\pi, \forall\, k \in I$

3. If $f(x)$ be an identity function, then

$$\lim_{x\to\infty}\left[\frac{f(x+1)f(x+2)+2}{f(x+3)f(x+4)}\right]^{4f(x)-2}$$ equals

(a) e^{-4} (b) e^{-16}

(c) e^{-12} (d) None of these

4. Define the sequence a_n recursively as follows: $a_0 = 0$, $a_1 = 1$ and for $n \geq 2$, a_n is the remainder of $a_{n-1} + a_{n-2}$ when divided by 3. The numerical value of $\sum_{k=2017}^{2024} a_k$ is

(a) 6 (b) 7 (c) 8 (d) 9

5. Given line $L : x + y = 9$, circle $M : 2x^2 + 2y^2 - 8x - 8y = 1$. Point A is on L; B, C are on M such that $\angle BAC = 45°$ and line AB passes through center of M. The range of x-coordinate of point A is

(a) [2, 5] (b) [3, 6] (c) [4, 7] (d) [5, 8]

6. Let a, b, c be real numbers and z_1, z_2, z_3 be complex numbers such that $|z_1| = |z_2| = |z_3| = 1$ and $\frac{z_1}{z_2}+\frac{z_2}{z_3}+\frac{z_3}{z_1} = 1$, then a possible value of $|az_1 + bz_2 + cz_3|$ is

(a) $\sqrt{(a+b)^2+c^2}$ (b) $\sqrt{(a-b)^2+c^2}$

(c) $\sqrt{(a-b)^2-c^2}$ (d) $\sqrt{(a+b)^2-c^2}$

More Than One Option Correct Type

7. If α is a root of $z^5 + z^3 + z + 3 = 0$, then

(a) $|\alpha| \geq 1$

(b) $|\alpha| < 1$

(c) α lies on or outside the unit circle $|z| = 1$

(d) α lies inside the unit circle $|z| = 1$

8. If $2\cos\theta = x + \frac{1}{x}$ and $2\cos\phi = y + \frac{1}{y}$, then

(a) $x^n + \frac{1}{x^n} = 2\cos(n\theta)$ (b) $\frac{x}{y}+\frac{y}{x} = 2\cos(\theta-\phi)$

(c) $xy + \frac{1}{xy} = 2\cos(\theta+\phi)$ (d) $|x| = |y| = 1$

9. $|z - i\,\mathrm{Re}(z)| = |z - \mathrm{Im}(z)|$, if

(a) $\mathrm{Im}(z) = 2, \mathrm{Re}(z) = 1$ (b) $\mathrm{Re}(z) = \mathrm{Im}(z)$

(c) $\mathrm{Re}(z) = 2, \mathrm{Im}(z) = 1$ (d) $\mathrm{Re}(z) + \mathrm{Im}(z) = 0$

10. A vector of magnitude 2 along a bisector of the angle between the two vectors $2\hat{i} - 2\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - 2\hat{k}$ is

(a) $\frac{2}{\sqrt{10}}(3\hat{i} - \hat{k})$ (b) $\frac{1}{\sqrt{26}}(\hat{i} - 4\hat{i} + 3\hat{k})$

(c) $\frac{2}{\sqrt{26}}(\hat{i} - 4\hat{i} + 3\hat{k})$ (d) $\frac{2}{\sqrt{5}}(\hat{i} - 3\hat{k})$

11. Let a plane passes through origin and is parallel to the line $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z+1}{-2}$ such that distance between plane and the line is 5/3. Then equation of the plane is

(a) $x - 2y + 2z = 0$ (b) $x - 2y - 2z = 0$

(c) $2x + 2y + z = 0$ (d) $x + y + z = 0$

Comprehension Type

Paragraph for Q. No. 12 and 13

The curve C has the equation $y=\dfrac{x}{\sqrt{x^2-2x+a}}$.

12. If $a > 1$, then C has

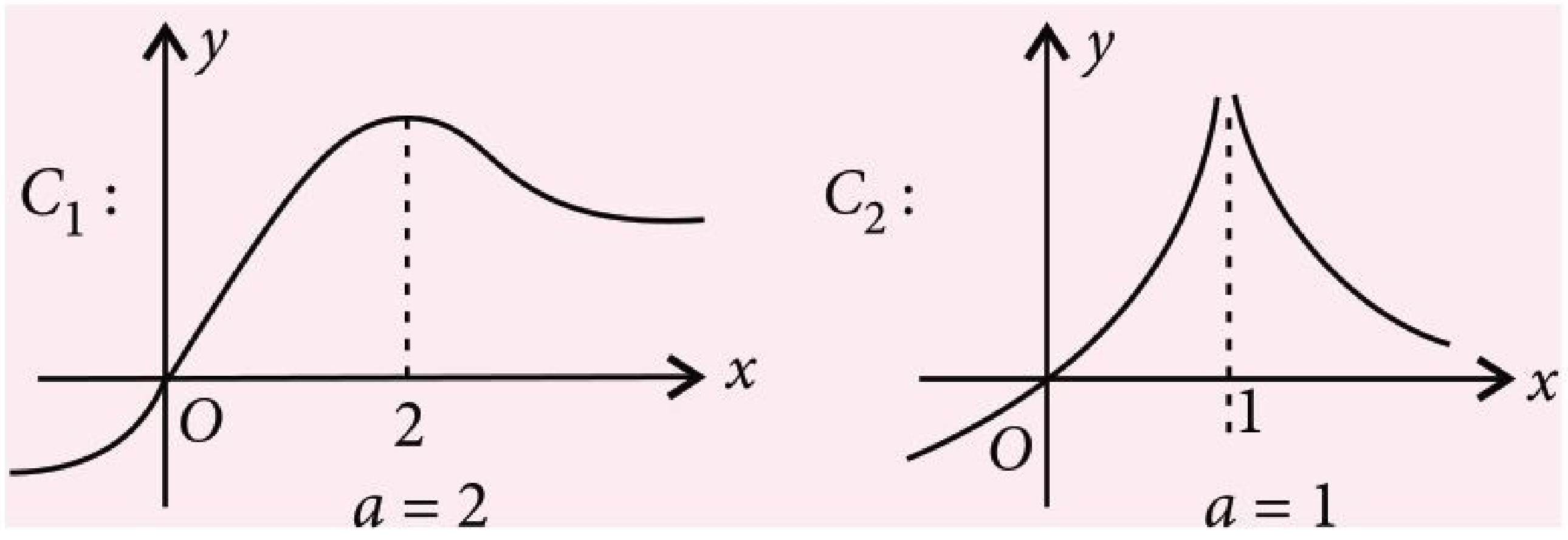
(a) exactly one stationary point

(b) exactly two stationary points

(c) no stationary point

(d) one or two stationary points depends on value of a.

13. Consider the two graphs C_1 and C_2 :



(a) Both C_1, C_2 are correct graphs

(b) Only C_1 is correct graph

(c) Only C_2 is correct graph

(d) Neither C_1 nor C_2 are correct graphs

Paragraph for Q. No. 14 and 15

Consider the differential equation

$xyp^2-(x^2-y^2)p-xy=0$, where $p=\dfrac{dy}{dx}$

14. A solution of the above differential equation passing through (2 , 2) is

(a) $x^2 + y^2 = 8$ (b) $y^2 = 2x$

(c) $xy = 4$ (d) $x^2y = 1$

15. The equations of orthogonal trajectories of the solutions of the given differential equation are

(a) $xy = c_1, x^2 - y^2 = c_2$ (b) $xy = c_1, x^2 + y^2 = c_2$

(c) $x^2 - y^2 = c_1, x^2 + y^2 = c_2$

(d) None of these

Matrix Match Type

16. Match the following :

	Column–I		Column–II
A.	The roots of cubic equation $(z+\alpha\beta)^3=\alpha^3, (\alpha\neq 0)$ represent the vertices of a triangle of area equal to	P.	$\lvert\tan\alpha\rvert$
B.	If α is a complex number, then the radius of the circle $\left\lvert\dfrac{z-\alpha}{z-\bar{\alpha}}\right\rvert=2$ is equal to	Q.	$\dfrac{3\sqrt{3}}{4}\lvert\alpha^2\rvert$
C.	If arg $(z)=\alpha$ and $\lvert z-1\rvert=1$, then $\left\lvert\dfrac{z-2}{z}\right\rvert$ is equal to	R.	$\dfrac{2}{3}\lvert\alpha-\bar{\alpha}\rvert$
D.	Let A and B represent complex numbers z_1 and z_2, which are roots of the equation $z^2+pz+q=0$. If $\angle AOB=\alpha\neq 0$ and $OA=OB$, where O is the origin, then p^2/q is equal to	S.	$4\cos^2\dfrac{\alpha}{2}$

Numerical Value Type

17. The number of solutions for the system of equations for which $|z| = 3$ and $|z+1-i|=\sqrt{2}$, equals ________.

18. Let $x, y\in\left[-\dfrac{\pi}{4},\dfrac{\pi}{4}\right]$ and $x^3 + \sin x = 2a$ and $4y^3+\dfrac{1}{2}\sin 2y+a=0$ then $\cos(x + 2y)$ = ________.

19. If $\dfrac{\tan 1}{\cos 2}+\dfrac{\tan 2}{\cos 4}+....+\dfrac{\tan 1024}{\cos 2048}=\tan\lambda-\tan\mu$, then $\left[\dfrac{\lambda-\mu}{1000}\right]$ ([·] denotes greatest integer function) = ________.

20. Let $f(x)$ be differentiable function in [2, 9]. If $f(2) = 3$ & $f'(x)\le 5\ \forall\ x\in(2, 9)$, then the maximum possible value of $f(x)$ at $x = 9$ is ________.

SOLUTIONS

1. (b) : Using the recurrence relation,

$$a_k = a_{k-1} + 4(k-1) + 3 = a_{k-2} + 4(k-2) + 4(k-1) + 2\cdot 3$$
$$= a_1 + 4(1 + 2 + 3 + ... + k - 1) + (k-1)\cdot 3$$
$$= (2k + 3)(k - 1)$$

So, $\lim\limits_{n\to\infty}\dfrac{\sqrt{a_{kn}}}{n}=\lim\limits_{n\to\infty}\sqrt{\left(2k+\dfrac{3}{n}\right)\left(k-\dfrac{1}{n}\right)}=\sqrt{2}\,k$

So, required limit $=\dfrac{1+4+4^2+...+4^{10}}{1+2+2^2+...+2^{10}}=\dfrac{4^{11}-1}{3(2^{11}-1)}$

$$=\frac{2^{11}+1}{3}=683$$

2. (a) : $x + y = \dfrac{\pi}{2}$ ...(i)

$\sin x + \sin y = \sqrt{2}$

$\Rightarrow\ 2\sin\dfrac{x+y}{2}\cos\dfrac{x-y}{2}=\sqrt{2}$

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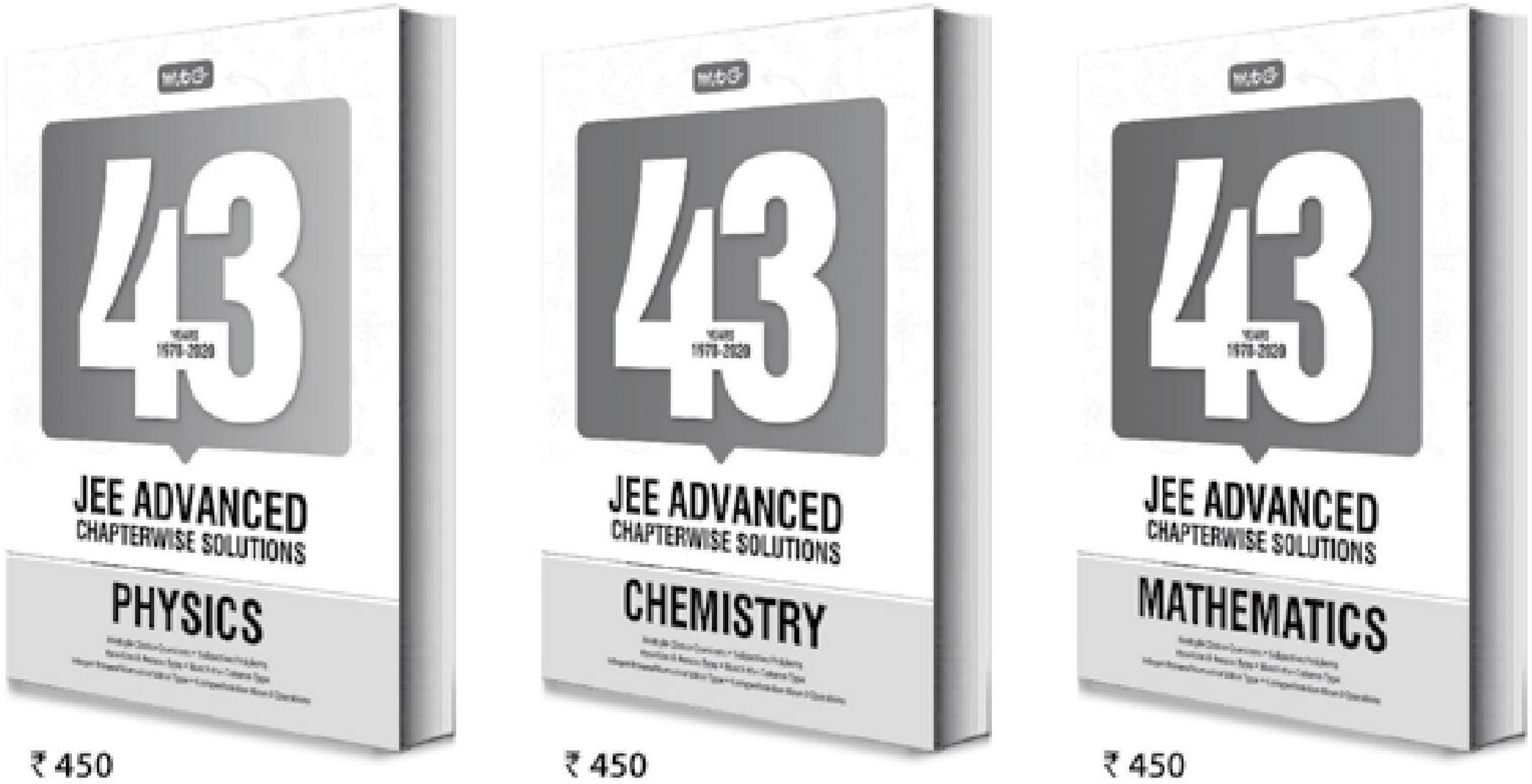
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$\Rightarrow \cos\frac{x-y}{2}=1$ (using (i))

$\Rightarrow x - y = 2\cdot 2\,k\pi = 4k\pi, k \in I$ ...(ii)

From (i) & (ii),

$x = 2k\pi + \frac{\pi}{4}, y = \frac{\pi}{4} - 2k\pi, k \in I$

3. **(b) :** As $f(x)$ is an identity function.

$\therefore\ f(x) = x \Rightarrow f(x+\alpha) = x + \alpha, \alpha \in R$

$$\therefore \lim_{x\to\infty}\left(\frac{f(x+1)f(x+2)+2}{f(x+3)f(x+4)}\right)^{4f(x)-2}$$

$$= \lim_{x\to\infty}\left(\frac{(x+1)(x+2)+2}{(x+3)(x+4)}\right)^{(4x-2)}$$

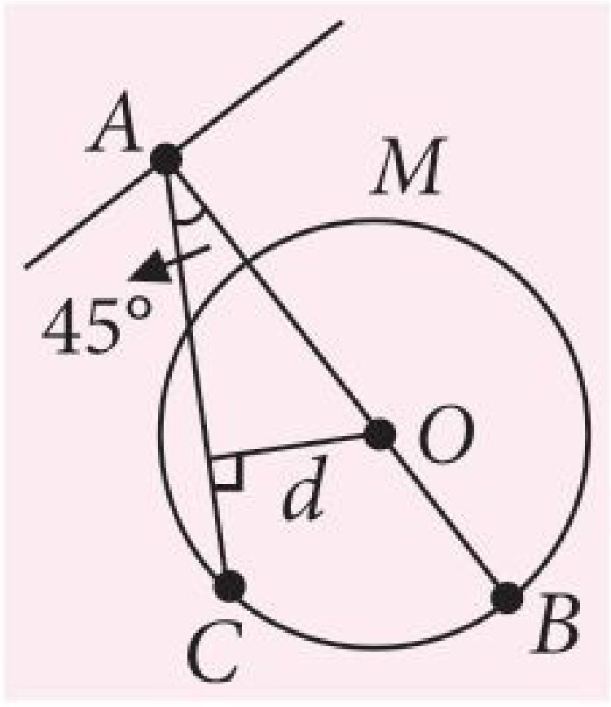
$$= \lim_{x\to\infty}\left(\frac{x^2+3x+4}{x^2+7x+12}\right)^{4x-2} \quad \text{(which is } 1^\infty \text{ form)}$$

$$= e^{\lim\limits_{x\to\infty}\frac{-(4x+8)(4x-2)}{x^2+7x+12}} = e^{-16}$$

4. **(d) :** The sequence written to some terms, looks like 0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, ... *i.e.*, $a_n = a_{n+8}$

Hence, $\sum\limits_{k=2017}^{2024} a_k = \sum\limits_{k=1}^{8} a_k = 9$

5. **(b) :** Let $A(t, 9-t)$



Then, $d = AO \sin 45°$

$= \frac{1}{\sqrt{2}}\cdot\sqrt{(t-2)^2+(9-t-2)^2}$

Now, $\frac{1}{\sqrt{2}}\sqrt{(t-2)^2+(9-t-2)^2} \le \sqrt{\frac{17}{2}}$

$\Rightarrow t^2 - 9t + 18 \le 0$ [$\because$ $d \le$ radius]

On solving, we get $3 \le t \le 6$

6. **(a) :** Let $\frac{z_1}{z_2} = e^{i\theta}, \frac{z_2}{z_3} = e^{i\alpha}$, then $\frac{z_3}{z_1} = e^{-i(\theta+\alpha)}$

$$\frac{z_1}{z_2}+\frac{z_2}{z_3}+\frac{z_3}{z_1} = 1 \Rightarrow \text{Im}\left(\frac{z_1}{z_2}\right)+\text{Im}\left(\frac{z_2}{z_3}\right)+\text{Im}\left(\frac{z_3}{z_1}\right)=0$$

i.e., $\sin\theta + \sin\alpha - \sin(\theta+\alpha) = 0$

Simplifying, $4\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\alpha}{2}\right)\sin\left(\frac{\theta+\alpha}{2}\right)=0$

It follows that $\theta = 2k\pi$ or $\alpha = 2k\pi$ or $\theta + \alpha = 2k\pi$, $(k \in I)$.

i.e., $z_1 = z_2$, $z_2 = z_3$ or $z_3 = z_1$.

If $z_1 = z_2$, then $\frac{z_1}{z_3}+\frac{z_3}{z_1} = 0 \Rightarrow \frac{z_3}{z_1} = \pm i$

$\Rightarrow |az_1 + bz_2 + cz_3| = |z_1||a + b \pm ci| = \sqrt{(a+b)^2+c^2}$

Other possible values are $\sqrt{(b+c)^2+a^2}$ and $\sqrt{(c+a)^2+b^2}$.

7. **(a, c) :** Suppose $|\alpha| < 1$, we have

$|-3| = |\alpha^5+\alpha^3+\alpha| \le |\alpha|^5+|\alpha|^3+|\alpha| < 1+1+1$

$\Rightarrow 3 < 3$, a contradiction

Thus $|\alpha| \ge 1$

$\Rightarrow \alpha$ lies either on or outside the unit circle $|z| = 1$.

8. **(a, b, c, d) :** We have, $2\cos\theta = x + \frac{1}{x}$

$\Rightarrow x = \cos\theta \pm i\sin\theta$

Similarly, $y = \cos\phi \pm i\sin\phi$

Thus, $|x| = |y| = 1$

$$\therefore x^n + \frac{1}{x^n} = \cos(n\theta)+i\sin(n\theta)+\cos(n\theta)-i\sin(n\theta)$$
$$= 2\cos(n\theta)$$

Now, $\frac{x}{y} = \cos(\theta-\phi)+i\sin(\theta-\phi)$

$\therefore \frac{x}{y}+\frac{y}{x} = 2\cos(\theta-\phi)$

And, $xy = \cos(\theta+\phi)+i\sin(\theta+\phi)$

$\Rightarrow xy + \frac{1}{xy} = 2\cos(\theta+\phi)$

9. **(b, d) :** Let $z = x + iy$,

then $|x + i(y - x)| = |x - y + iy| \Rightarrow x^2 = y^2 \Rightarrow x = \pm y$

10. **(a, c) :** $\vec{a} = 2\hat{i} - 2\hat{j} + \hat{k}, \vec{b} = \hat{i} + 2\hat{j} - 2\hat{k}$

$\therefore\ |\vec{a}| = 3, |\vec{b}| = 3$

$\therefore$ A vector along bisectors

$= \frac{\vec{a}}{|\vec{a}|} \pm \frac{\vec{b}}{|\vec{b}|} = \hat{i} - \frac{1}{3}\hat{k}$ or $\frac{1}{3}\hat{i} - \frac{4}{3}\hat{j} + \hat{k}$

$\therefore$ The required vector

$$= 2.\frac{\hat{i}-\frac{1}{3}\hat{k}}{\sqrt{1^2+\left(-\frac{1}{3}\right)^2}} \text{ or } 2.\frac{\frac{1}{3}\hat{i}-\frac{4}{3}\hat{j}+\hat{k}}{\sqrt{\left(\frac{1}{3}\right)^2+\left(-\frac{4}{3}\right)^2+1^2}}$$

$$= \frac{2}{\sqrt{10}}(3\hat{i}-\hat{k}) \text{ or } \frac{2}{\sqrt{26}}(\hat{i}-4\hat{j}+3\hat{k})$$

11. **(a, c) :** Let the equation of plane be
$lx + my + nz = 0$, where l, m, n be d.c.'s

$\Rightarrow l^2 + m^2 + n^2 = 1$ ...(i)

Given line is $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z+1}{-2}$

$\therefore\ 2l - m - 2n = 0$ ...(ii)

Also, $\frac{l-3m-n}{\sqrt{l^2+m^2+n^2}}=\frac{5}{3} \Rightarrow l-3m-n=\frac{5}{3}$ ...(iii)

Solving (i), (ii) and (iii) we get equation of plane as
$x-2y+2z=0$ or $2x+2y+z=0$.

(12-13) : $\frac{dy}{dx}=\frac{(a-x)}{(x^2-2x+a)^{3/2}}=0$ only at $x=a$

Exactly one stationary point.

And for graph of C_1, $a=2$, has stationary point at $(2, \sqrt{2})$ and $x \to \infty, y \to 1$ and $x \to -\infty, y \to -1$.

For $a=1$, $y=\frac{x}{|x-1|}$, a vertical asymptote at $x=1$.

12. (a) **13. (a)**

(14 – 15):

$xyp^2 - x^2p + y^2p - xy = 0$

$\Rightarrow xp(yp-x)+y(yp-x)=0 \Rightarrow (yp-x)(xp+y)=0$

$\Rightarrow p=\frac{x}{y}$ or $p=\frac{-y}{x} \Rightarrow \frac{dy}{dx}=\frac{x}{y}$ or $\frac{dy}{dx}=\frac{-y}{x}$

$\Rightarrow \int y\,dy=\int x\,dx$ or $x\,dy+y\,dx=0$

$\Rightarrow \frac{y^2}{2}=\frac{x^2}{2}+c$ or $\int d(xy)=0$

$\Rightarrow y^2-x^2=c_1$ or $xy=c_2$

14. (c) **15. (a)**

16. A → Q; B → R; C → P; D → S

(A) $(z+\alpha\beta)^3=\alpha^3 \Rightarrow z+\alpha\beta=\alpha, \omega\alpha, \omega^2\alpha$

$\Rightarrow z=\alpha-\alpha\beta, \omega\alpha-\alpha\beta, \omega^2\alpha-\alpha\beta$,

say z_1, z_2, z_3 respectively

Now, $|z_1-z_2|=|z_2-z_3|=|z_3-z_1|=\sqrt{3}|\alpha|$.

So, the triangle is equilateral.

$\therefore$ area of the triangle $=\frac{\sqrt{3}}{4}|z_1-z_2|^2=\frac{3\sqrt{3}}{4}|\alpha|^2$

(B) $|z-\alpha|^2=4|z-\bar{\alpha}|^2$

$\Rightarrow z\bar{z}-\vec{\alpha}z-\alpha\bar{z}+\alpha\bar{\alpha}=4(z\bar{z}-\alpha z-\bar{\alpha}\bar{z}+\alpha\bar{\alpha})$

$\Rightarrow 3z\bar{z}+(\alpha-4\bar{\alpha})\bar{z}+(\bar{\alpha}-4\alpha)z+3\alpha\bar{\alpha}=0$

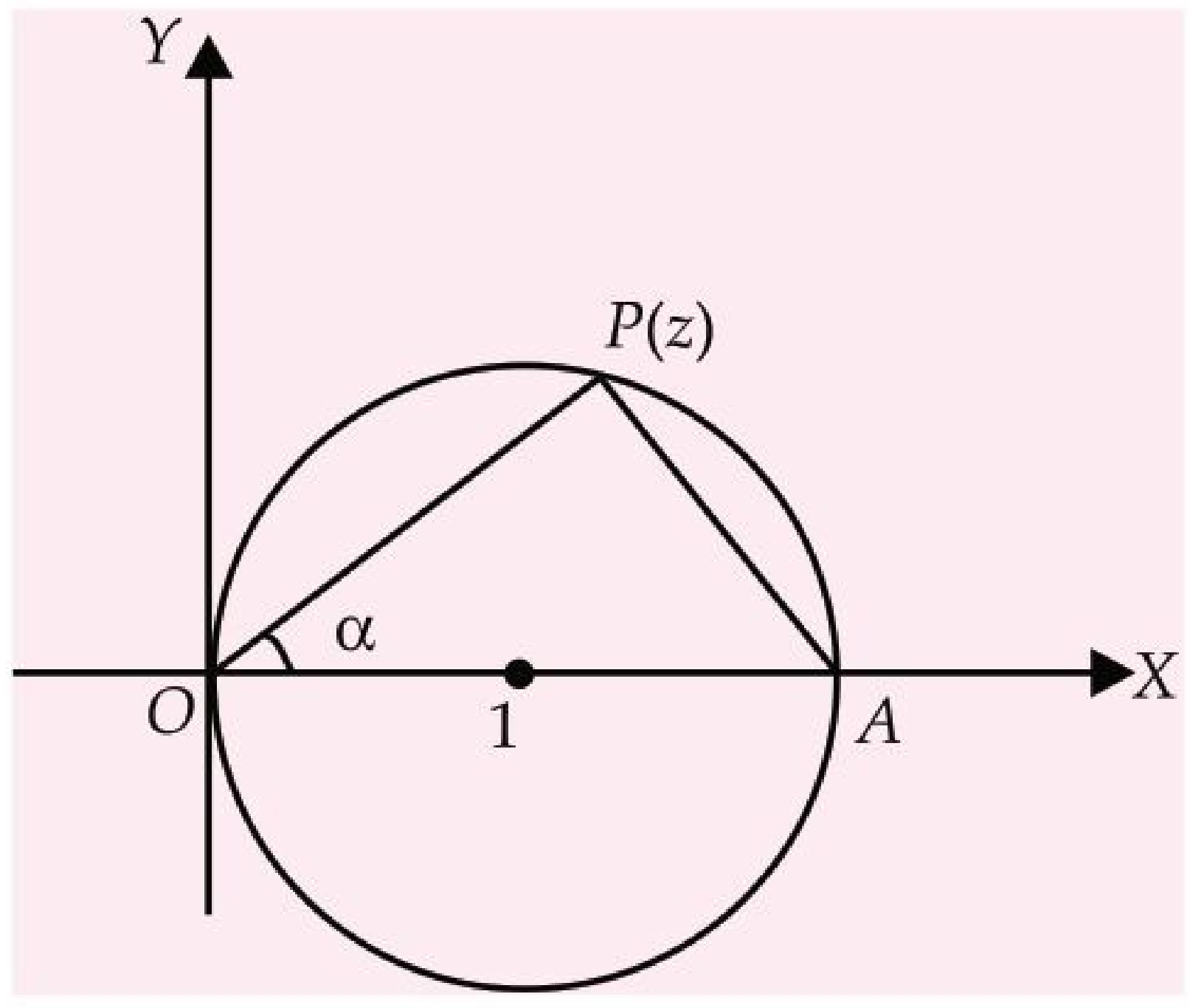
$\Rightarrow z\bar{z}+\frac{\alpha-4\bar{\alpha}}{3}\bar{z}+\frac{\bar{\alpha}-4\alpha}{3}z+\alpha\bar{\alpha}=0$,

which is a circle of radius

$=\sqrt{\left|\frac{\alpha-4\bar{\alpha}}{3}\right|^2-\alpha\bar{\alpha}}=\sqrt{-\frac{4}{9}(\alpha-\bar{\alpha})^2}=\frac{2}{3}|\alpha-\bar{\alpha}|$.

(C) z lies on a circle of radius 1 and centre at $(1, 0)$

$\angle OPA=\pm\frac{\pi}{2} \Rightarrow \frac{2-z}{0-z}=\frac{|2-z|}{z}e^{\pm i\frac{\pi}{2}}$



$\Rightarrow \frac{z-2}{z}=\frac{AP}{OP}(\pm i)=\pm i\tan\alpha \quad \therefore \left|\frac{z-2}{z}\right|=|\tan\alpha|$

(D) We have, $z_1+z_2=-p$ and $z_1z_2=q$

Also, $\frac{z_2}{z_1}=\cos\alpha \pm i\sin\alpha \Rightarrow \frac{z_2-z_1\cos\alpha}{z_1}=\pm i\sin\alpha$

$\Rightarrow z_2^2-2z_2z_1\cos\alpha+z_1^2\cos^2\alpha=-z_1^2\sin^2\alpha$

$\Rightarrow z_1^2+z_2^2=2z_1z_2\cos\alpha$

$\therefore (z_1+z_2)^2=2z_1z_2(1+\cos\alpha) \Rightarrow \frac{p^2}{q}=4\cos^2\frac{\alpha}{2}$.

17. (0) : Let $z=x+iy$

$\therefore |z|=3 \Rightarrow x^2+y^2=9$ (represents a circle)

and $(x+1)^2+(y-1)^2=2$ (also a circle)

$\therefore C_1(0, 0)$, $r_1=3$ and $C_2(-1, 1)$, $r_2=\sqrt{2}$

$\therefore C_1C_2=\sqrt{2}$ and $r_1-r_2=3-\sqrt{2}$,

Now, $C_1C_2 < r_1-r_2$ as $r_2=\sqrt{2}=1.414$ and

$r_1-r_2=3-1.414=1.586$ (approx.)

$\therefore$ Circles are non intersecting and one lies completely with in the other.

$\therefore$ The circles don't have any common point.

$\Rightarrow$ Number of solutions = 0

18. (1) : Replacing $2y$ with z and taking $f(x)=x^3+\sin x$, the given equation becomes

$f(x)=f(-z) \Rightarrow x=-z$ as $f(x)$ is odd

and increasing in $\left[\frac{-\pi}{4}, \frac{\pi}{4}\right]$

So, $x=-2y$ $\therefore \cos(x+2y)=\cos(0)=1$

19. (2) : Using, $\frac{\tan\theta}{\cos 2\theta}=\tan 2\theta-\tan\theta$, the given sum becomes

$(\tan 2-\tan 1)+(\tan 4-\tan 2)+....+(\tan 2048-\tan 1024)$

$=\tan 2048-\tan 1=\tan\lambda-\tan\mu \therefore \left[\frac{\lambda-\mu}{1000}\right]=\left[\frac{2047}{1000}\right]=2$

20. (38) : Given, $f'(x) \le 5$

$\Rightarrow \int_2^9 f'(x)dx \le \int_2^9 5dx \Rightarrow |f(x)|_2^9 \le [5x]_2^9$

$\Rightarrow f(9)-f(2) \le 45-10=35 \Rightarrow f(9) \le 35+f(2)$

$\Rightarrow f(9) \le 38 \quad (\because f(2)=3)$

$\therefore$ Maximum possible value of $f(x)$ at $x=9$ is 38.

◆

PRACTICE PAPER 2022
JEE MAIN

Time : 1 hr.

Series - 1

The entire syllabus of Mathematics of JEE MAIN is being divided into four parts. Each part contains 4 units. There will be a Practice Paper which will be published in the subsequent issues. The syllabus for module break-up is given below:

Units	Topic	Syllabus In Detail
I	Sets, Relations and Functions	Sets and their representation; Union, intersection and complement of sets and their algebraic properties; Power set; Relation, Type of relations, equivalence relations, Functions; one-one, into and onto functions, the composition of functions.
II	Complex Numbers and Quadratic Equations	Complex numbers as ordered pairs of reals, Representation of complex numbers in the form *a* + *ib* and their representation in a plane, Argand diagram, algebra of complex number, modulus and argument (or amplitude) of a complex number, square root of a complex number, triangle inequality, Quadratic equations in real and complex number system and their solutions, Relations between roots and coefficient, nature of roots, the formation of quadratic equations with given roots.
III	Matrices and Determinants	Matrices, algebra of matrices, type of matrices, determinants and matrices of order two and three, properties of determinants, evaluation of determinants, area of triangles using determinants, Adjoint and evaluation of inverse of a square matrix using determinants and elementary transformations, Test of consistency and solution of simultaneous linear equations in two or three variables using determinants and matrices.
IV	Permutations and Combinations	The fundamental principle of counting, permutation as an arrangement and combination as section, Meaning of *P* (*n*,*r*) and *C* (*n*,*r*), simple applications.

SINGLE OPTION CORRECT TYPE

1. Let $f : (4, 6) \to (6, 8)$ be a function defined by $f(x) = x + \left[\frac{x}{2}\right]$ (where $[\cdot]$ denotes the greatest integer function), then $f^{-1}(x)$ is equal to

(a) $x - \left[\frac{x}{2}\right]$ (b) $-x - 2$

(c) $x - 2$ (d) $\dfrac{1}{x + \left[\frac{\pi}{6}\right]}$

2. Let A and B be two sets such that $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X. Then,

(a) $A = B$ (b) $A = X$

(c) $B = X$ (d) $A \cup B = X$

3. $\sin^{-1}\left[\frac{1}{i}(z-1)\right]$, where z is non-real, can be the angle of a triangle if

(a) $\text{Re}(z) = 1, \text{Im}(z) = 2$

(b) $\text{Re}(z) = 1, 0 < \text{Im}(z) \le 1$

(c) $\text{Re}(z) + \text{Im}(z) = 0$

(d) $\text{Re}(z) = \text{Im}(z)$

4. If z_1 and z_2 are two of the 8th roots of unity such that $\arg\left(\frac{z_1}{z_2}\right)$ is least positive, then $\frac{z_1}{z_2}$ is

(a) $1 + i$ (b) $1 - i$

(c) $\frac{1+i}{\sqrt{2}}$ (d) $\frac{1-i}{\sqrt{2}}$

5. If $f: R\times R \to R$ such that $f(x+iy)=\sqrt{x^2+y^2}$, then f is

(a) many-one into (b) one-one and onto

(c) many-one and onto (d) one-one and into.

6. On the set N of natural numbers, define the relation R by aRb iff the G.C.D. of a and b is 2, then R is

(a) reflexive but not symmetric

(b) symmetric only

(c) equivalence

(d) transitive only

7. Point at which the curves $\arg(z-3i)=\frac{3\pi}{4}$ and $\arg(2z+1-2i)=\frac{\pi}{4}$ intersect, is

(a) $\left(\frac{3}{4},\frac{9}{4}\right)$ (b) $\left(\frac{3}{4},\frac{7}{4}\right)$

(c) $\left(\frac{2}{4},\frac{7}{4}\right)$ (d) $\left(\frac{6}{7},\frac{4}{7}\right)$

8. If $D_k=\begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n+1 \end{vmatrix}$ and $\sum_{k=1}^{n} D_k = 72,$ then value of n equals

(a) 4 (b) 6 (c) 7 (d) 8

9. From the matrix equation $AB = AC$, we say $B = C$, provided

(a) A is singular (b) A is square matrix

(c) A is skew-symmetric (d) A is non-singular

10. For any three sets A_1, A_2, A_3, let $B_1 = A_1$, $B_2 = A_2 - A_1$ and $B_3 = A_3 - (A_1 \cup A_2)$, then

(a) $A_1 \cup A_2 \cup A_3 \supset B_1 \cup B_2 \cup B_3$

(b) $A_1 \cup A_2 \cup A_3 = B_1 \cup B_2 \cup B_3$

(c) $A_1 \cup A_2 \cup A_3 \subset B_1 \cup B_2 \cup B_3$

(d) None of these

11. Let $z = \cos\theta + i\sin\theta$. Then, the values of $\sum_{m=1}^{15} \operatorname{Im}(z^{2m-1})$ at $\theta = 2°$, is

(a) $\frac{1}{\sin 2°}$ (b) $\frac{1}{3\sin 2°}$

(c) $\frac{1}{2\sin 2°}$ (d) $\frac{1}{4\sin 2°}$

12. Let α, β be the roots of the equation $x^2 - x + p = 0$ and γ, δ be the roots of the equation $x^2 - 4x + q = 0$. If $\alpha, \beta, \gamma, \delta$ are in G.P., then the integral values of p and q respectively, are

(a) $-2, -32$ (b) $-2, 3$

(c) $-6, 3$ (d) $-6, -32$

13. If $f(x)=\begin{vmatrix} x & \sin x & \cos x \\ x^2 & \tan x & -x^3 \\ 2x & \sin 2x & 5x \end{vmatrix}$, then $\lim_{x\to 0} \frac{f'(x)}{x}$ is

(a) 1 (b) 4 (c) 3 (d) -4

14. If a square matrix $A = [a_{ij}]$, $a_{ij} = i^2 - j^2$ is of even order then

(a) A is a skew-symmetric and $|A| = 0$

(b) A is symmetric and $|A|$ is a perfect square

(c) A is symmetric and $|A| = 0$

(d) None of these

15. A test consists of 6 multiple choice questions each having 4 alternative answers of which only one is correct. Also only one of the alternatives must be marked by each candidate. The number of ways of getting exactly 4 correct answers by a candidate answering all the questions is

(a) $4^6 - 3^2$ (b) 135

(c) 55 (d) 120

16. The number of positive terms in the sequence $x_n = \frac{195}{4\cdot {}^nP_n} - \frac{{}^{n+3}P_3}{{}^{n+1}P_{n+1}}$, $n \in N$ is

(a) 2 (b) 3

(c) 4 (d) None of these

17. The number of ordered triplets of positive integers which are solutions of the equation $x + y + z = 100$ is

(a) 6005 (b) 4851

(c) 5081 (d) 5151

18. The total number of ways in which 5 balls of different colours can be distributed among 3 persons, so that each person get at least one ball, is

(a) 75 (b) 144

(c) 210 (d) 243

19. If ${}^{n-1}C_r = (k^2 - 3)\,{}^nC_{r+1}$, then $k \in$

(a) $(-\infty, -2]$ (b) $[2, \infty)$

(c) $(-\sqrt{3}, \sqrt{3})$ (d) $(\sqrt{3}, 2]$

20. If $f: A \to B$ given by $3^{f(x)} + 2^{-x} = 4$ is a bijection, then

(a) $A = \{x \in R : -1 < x < \infty\}$, $B = \{x \in R : 2 < x < 4\}$

(b) $A = \{x \in R : -3 < x < \infty\}$, $B = \{x \in R : 0 < x < 4\}$

(c) $A = \{x \in R : -2 < x < \infty\}$, $B = \{x \in R : 0 < x < 4\}$

(d) None of these

NUMERICAL ANSWER TYPE

21. If $x^2 - x + 1 = 0$, then the value of $\sum_{n=1}^{5}\left(x^n + \frac{1}{x^n}\right)^2$ is ______.

22. The least positive integral value of n for which $(\sqrt{3}+i)^n = (\sqrt{3}-i)^n$, is ______.

23. If $f(x) = \begin{vmatrix} 3 & 3x & 3x^2+2a^2 \\ 3x & 3x^2+2a^2 & 3x^3+6a^2x \\ 3x^2+2a^2 & 3x^3+6a^2x & 3x^4+12a^2x^2+2a^4 \end{vmatrix}$,

then $\int_0^2 f(x)dx = ka^6$, where k is equal to ______.

24. If $0 \le [x] < 2, -1 \le [y] < 1$ and $1 \le [z] < 3$, ($[\cdot]$ denotes the greatest integer function) then the maximum value of determinant $\Delta = \begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$ is ______.

25. If $A = \begin{pmatrix} i & -i \\ -i & i \end{pmatrix}$ (where $i = \sqrt{-1}$) and $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$ then $A^4 = kB$. The value of k is_____.

26. For the set of linear equations

$\lambda x - 3y + z = 0$

$x + \lambda y + 3z = 1$

$3x + y + 5z = 2$

the values of λ, for which the equations does not have a unique solution are $\left(\frac{a}{5}, b\right)$, then $a + b$ equals______.

27. The number of numbers greater than 50000 that can be formed by using the digits 3, 5, 6, 6, 7 is_____.

28. The rank of the word 'FLOWER' is ______.

29. The letters of the word COCHIN are permuted and all the permutations are arranged in an alphabetical order as in English dictionary. The number of words that appear before the word COCHIN, is ______.

30. If the real valued function $f(x) = \frac{a^x - 1}{x^n(a^x+1)}$ is even, then least positive integral value of n is equal to ______.

SOLUTIONS

1. **(c) :** Here, $4 < x < 6 \Rightarrow 2 < \frac{x}{2} < 3 \therefore \left[\frac{x}{2}\right] = 2$

So, $f(x) = x + \left[\frac{x}{2}\right] = x + 2, x \in (4, 6)$

Let for $f(x) = x + \left[\frac{x}{2}\right] = x + 2$, $x \in (4, 6)$, there exists $y \in (6, 8)$ such that $f(x) = y$

$\Rightarrow \ y = x + 2 \Rightarrow x = y - 2$

$\therefore \ f^{-1}(y) = y - 2$

$\Rightarrow \ f^{-1}(x) = x - 2$

2. **(a) :** We have, $A \cup X = B \cup X$ for some set X

$\Rightarrow \ A \cap (A \cup X) = A \cap (B \cup X)$

$\Rightarrow \ (A \cap A) \cup (A \cap X) = (A \cap B) \cup (A \cap X)$

$\Rightarrow \ A \cup \phi = (A \cap B) \cup \phi \quad [\because A \cap X = \phi]$

$\Rightarrow \ A = (A \cap B)$ ...(i)

Again, $A \cup X = B \cup X$

$\Rightarrow \ B \cap (A \cup X) = B \cap (B \cup X)$

$\Rightarrow \ (B \cap A) \cup (B \cap X) = (B \cap B) \cup (B \cap X)$

$\Rightarrow \ (B \cap A) \cup \phi = B \cup \phi \quad [\because B \cap X = \phi]$

$\Rightarrow \ B = (A \cap B)$ ...(ii)

From (i) and (ii), we have $A = B$.

3. **(b):** $\sin^{-1}\left(\frac{z-1}{i}\right)$ will be an angle of a triangle if $\frac{z-1}{i}$ is real.

Let $z = x + iy$, therefore

$\frac{z-1}{i} = \frac{x+iy-1}{i} = \frac{i(x+iy-1)}{-1} = y - i(x-1)$ is real

$\Rightarrow \ x - 1 = 0$ *i.e.* $x = 1$

$\therefore \ \sin^{-1}\frac{z-1}{i} = \sin^{-1} y \quad \therefore 0 < y \le 1$

4. **(c) :** We know that the nth roots of unity lies on the circle $|z| = 1$ and divide its circumference into n equal parts. Also, nth roots of unity are given by

$z_r = e^{\frac{i2r\pi}{n}}; r = 0, 1, 2, ..., (n-1)$

$\therefore$ 8th roots of unity are given by

$z_r = e^{\frac{ir\pi}{4}}; r = 0, 1, 2, ..., 7$

We know that $\arg\left(\frac{z_1}{z_2}\right) = \arg(z_1) - \arg(z_2)$.

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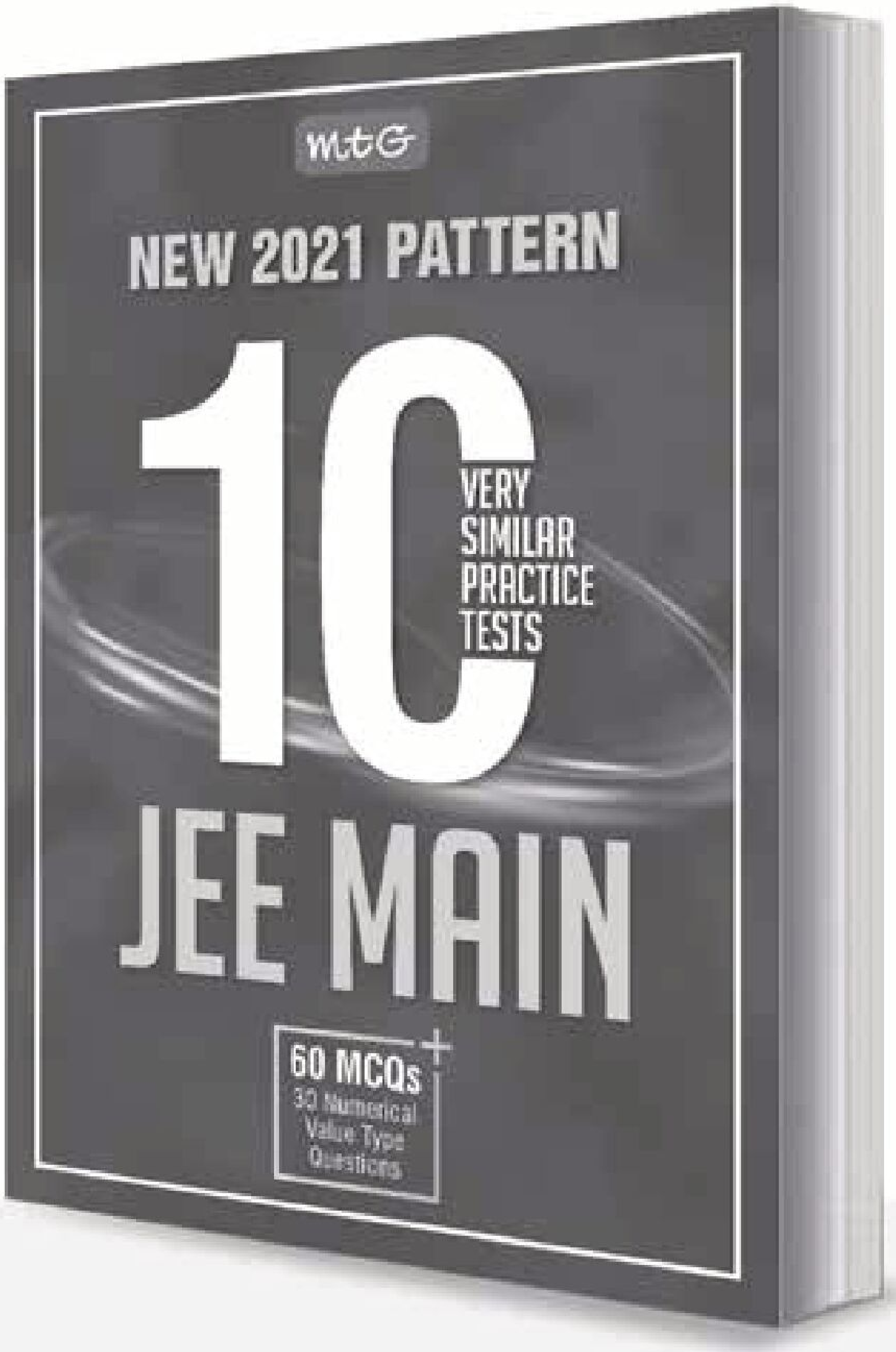
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Therefore, $\arg\left(\frac{z_1}{z_2}\right)$ will have least positive value, if z_1 and z_2 represent any two consecutive points on the circle $|z| = 1$.

Let $z_1 = z_r = e^{\frac{ir\pi}{4}}$.

Thus, $z_2 = z_{r-1} = e^{\frac{i(r-1)\pi}{4}}$

$$\therefore \quad \left(\frac{z_1}{z_2}\right) = e^{i\pi/4} = \cos\frac{\pi}{4} + i\sin\frac{\pi}{4} = \frac{1+i}{\sqrt{2}}.$$

5. (a) : Since $f(x + iy) = f(x - iy)$ $\therefore$ f is many one.
Also range $\in R^+$ and co-domain $\in R$
$\therefore$ range $\subset$ co-domain $\Rightarrow f$ is into
Hence f is many-one into.

6. (b): For any $a \in N$, we have G.C.D.$(a, a) = a$.
Thus R is not reflexive.
Let $(a, b) \in R$. Then aRb
$\Rightarrow$ G.C.D.$(a, b) = 2 =$ G.C.D. (b, a)
$\Rightarrow$ bRa. So, R is symmetric.
We observed that G.C.D.(6, 4) = 2
and G.C.D.(4, 18) = 2. But, G.C.D.(6, 18) = 6.
Hence, $(a, b) \in R$, $(b, c) \in R$
$\Rightarrow$ $(a, c) \in R$ is not satisfied. So, R is not transitive.

7. (a) : Let $z = x + iy$

$\therefore \quad z - 3i = x + iy - 3i = x + i(y - 3)$

So, $\arg(x + i(y - 3)) = \frac{3\pi}{4}$

$$\Rightarrow \quad \tan^{-1}\left(\frac{y-3}{x}\right) = \frac{3\pi}{4} \Rightarrow x + y = 3 \qquad \text{...(i)}$$

Again, $2z + 1 - 2i = 2(x + iy) + 1 - 2i = 2x + 1 + i(2y - 2)$

$$\therefore \quad \arg(2z + 1 - 2i) = \frac{\pi}{4} \Rightarrow \arg\{(2x+1) + i(2y-2)\} = \frac{\pi}{4}$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{2y-2}{2x+1}\right) = \frac{\pi}{4} \quad \Rightarrow \quad \frac{2y-2}{2x+1} = 1$$

$$\Rightarrow \quad 2x - 2y = -3 \qquad \text{...(ii)}$$

Solving (i) and (ii) we get, $x = \frac{3}{4}, y = \frac{9}{4}$

8. (d): Given,

$$D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n+1 \end{vmatrix} \text{ and } \sum_{k=1}^{n} D_k = 72$$

$$\therefore \quad \sum_{k=1}^{n} D_k = \begin{vmatrix} \sum_{k=1}^{n} 1 & \sum_{k=1}^{n} 2k & \sum_{k=1}^{n} (2k-1) \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n+1 \end{vmatrix}$$

$$= \begin{vmatrix} n & n(n+1) & n^2 \\ n & n^2+n+1 & n^2 \\ n & n^2+n & n^2+n+1 \end{vmatrix}$$

Applying $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$, we get

$$\begin{vmatrix} n & n(n+1) & n^2 \\ 0 & 1 & 0 \\ 0 & 0 & n+1 \end{vmatrix} = n(n+1)$$

Hence, $n(n + 1) = 72 = 8(8 + 1) \Rightarrow n = 8$

9. (d): Let $|A| \neq 0 \quad \therefore A^{-1}$ exist
Given that $AB = AC$
$\therefore \quad A^{-1}(AB) = A^{-1}(AC)$
$\Rightarrow \quad B = C$ (By associative property)
$\Rightarrow$ A is non-singular.

10. (b): $B_1 \cup B_2 = A_1 \cup (A_2 - A_1) = A_1 \cup (A_2 \cap A_1')$
$= (A_1 \cup A_2) \cap (A_1 \cup A_1') = (A_1 \cup A_2) \cap U = (A_1 \cup A_2)$
Now, $B_1 \cup B_2 \cup B_3 = (B_1 \cup B_2) \cup B_3$
$= (A_1 \cup A_2) \cup [A_3 - (A_1 \cup A_2)]$
$= (A_1 \cup A_2) \cup [A_3 \cap (A_1 \cup A_2)']$
$= [(A_1 \cup A_2) \cup A_3] \cap [(A_1 \cup A_2) \cup (A_1 \cup A_2)']$
$= [A_1 \cup A_2 \cup A_3] \cap U = A_1 \cup A_2 \cup A_3$

11. (d): We have, $z = \cos\theta + i\sin\theta$

$$\Rightarrow \quad z^{2m-1} = \cos(2m-1)\theta + i\sin(2m-1)\theta$$

$$\text{Now, } \sum_{m=1}^{15} \text{Im}(z^{2m-1}) = \sum_{m=1}^{15} \sin(2m-1)\theta$$

$$= \frac{\sin(\theta + 14\theta)\sin 15\theta}{\sin\theta} = \frac{\sin^2 15\theta}{\sin\theta}$$

$\therefore$ At $\theta = 2°$, we have

$$\therefore \quad \sum_{m=1}^{15} \text{Im}(z^{2m-1}) = \frac{1 - \cos 60°}{2\sin 2°} = \frac{1}{4\sin 2°}$$

12. (a) : Given, α, β are the roots of the equation

$x^2 - x + p = 0$

$\therefore \quad \alpha + \beta = 1, \alpha\beta = p$

Also given that γ and δ are the roots of the equation

$x^2 - 4x + q = 0$

$\therefore \quad \gamma + \delta = 4, \gamma\delta = q$

Let r be the common ratio of the G.P. $\alpha, \beta, \gamma, \delta$.

$\therefore \quad \beta = \alpha r, \gamma = \alpha r^2$ and $\delta = \alpha r^3$

Now, $\alpha+\beta=1 \Rightarrow \alpha+\alpha r=1$

$\gamma+\delta=4 \Rightarrow \alpha r^2+\alpha r^3=4$

$$\therefore \quad \frac{\alpha r^2+\alpha r^3}{\alpha+\alpha r}=4 \Rightarrow r^2=4 \Rightarrow r=\pm 2$$

When $r = 2$

$$\alpha+\beta=1 \Rightarrow \alpha+\alpha r=1 \Rightarrow 3\alpha=1 \Rightarrow \alpha=\frac{1}{3}$$

Now, $p=\alpha\beta=\alpha^2 r=\dfrac{2}{9}$

But, p is an integer.

Therefore, $r = 2$ is not possible.

When $r = -2$

$\alpha+\beta=1 \Rightarrow \alpha+\alpha r=1 \Rightarrow -\alpha=1 \Rightarrow \alpha=-1$

$\therefore \quad \alpha+\beta=1 \Rightarrow \beta=2$

Now, $p=\alpha\beta \Rightarrow p=-2$ and $q=\gamma\delta=(\alpha r^2)(\alpha r^3)=-32$

Hence, $p = -2$ and $q = -32$

13. (d) : $f'(x)=\begin{vmatrix} 1 & \cos x & -\sin x \\ x^2 & \tan x & -x^3 \\ 2x & \sin 2x & 5x \end{vmatrix}$

$$+\begin{vmatrix} x & \sin x & \cos x \\ 2x & \sec^2 x & -3x^2 \\ 2x & \sin 2x & 5x \end{vmatrix}+\begin{vmatrix} x & \sin x & \cos x \\ x^2 & \tan x & -x^3 \\ 2 & 2\cos 2x & 5 \end{vmatrix}$$

Applying $R_2 \to \frac{1}{x}R_2, R_3 \to \frac{1}{x}R_3$ and $R_2 \to \frac{1}{x}R_2$ on 1st, 2nd and 3rd determinants respectively, we get

$$\frac{f'(x)}{x}=\begin{vmatrix} 1 & \cos x & -\sin x \\ x & \dfrac{\tan x}{x} & -x^2 \\ 2x & \sin 2x & 5x \end{vmatrix}+\begin{vmatrix} x & \sin x & \cos x \\ 2x & \sec^2 x & -3x^2 \\ 2 & \dfrac{\sin 2x}{x} & 5 \end{vmatrix}$$

$$+\begin{vmatrix} x & \sin x & \cos x \\ x & \dfrac{\tan x}{x} & -x^2 \\ 2 & 2\cos 2x & 5 \end{vmatrix}$$

$$\therefore \lim_{x\to 0}\frac{f'(x)}{x}=\begin{vmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{vmatrix}+\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 2 & 5 \end{vmatrix}+\begin{vmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 2 & 2 & 5 \end{vmatrix}=-4$$

14. (d): We have, $A=[a_{ij}], a_{ij}=i^2-j^2$

$\therefore \quad A=\begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$ and $|A| = 9$, a perfect square.

15. (b): The number of ways by which 4 questions with correct answers can be chosen in ${}^6C_4$. Since only one of the four alternatives is correct, the wrong answers can be given in 3 different ways for each of the two remaining questions attempted unsuccessfully by the candidate. The desired number of ways = ${}^6C_4 \cdot 3^2 = 135$

16. (c) : Given that $x_n=\dfrac{195}{4\cdot{}^nP_n}-\dfrac{{}^{n+3}P_3}{{}^{n+1}P_{n+1}},\ n\in N$

$$=\frac{195}{4\cdot n!}-\frac{(n+3)(n+2)(n+1)}{(n+1)!}=\frac{195}{4\cdot n!}-\frac{(n+3)(n+2)}{n!}$$

$$=\frac{195-4n^2-20n-24}{4n!}=\frac{171-4n^2-20n}{4\cdot n!}$$

$\because\ x_n$ is positive $\therefore\ \dfrac{171-4n^2-20n}{4\cdot n!}>0$

$\Rightarrow\ 4n^2+20n-171<0$

The above inequality is true for n = 1, 2, 3 and 4.

17. (b): The number of solutions of the given equation is same as the number of ways of distributing 100 items among 3 persons such that each person can receive at least one items.

SAMURAI SUDOKU

ANSWER · SEPTEMBER 2021

8	4	6	5	3	9	1	2	7				2	6	4	7	9	5	8	1	3
7	1	5	6	2	4	8	9	3				7	1	8	3	6	2	9	4	5
3	9	2	8	1	7	4	5	6				5	9	3	1	4	8	7	2	6
6	3	7	4	9	2	5	1	8				1	3	6	8	2	9	5	7	4
4	2	1	7	5	8	3	6	9				9	2	5	4	3	7	6	8	1
9	5	8	1	6	3	7	4	2				4	8	7	6	5	1	3	9	2
1	6	3	2	7	5	9	8	4	7	3	2	6	5	1	9	8	4	2	3	7
2	7	4	9	8	1	6	3	5	1	9	4	8	7	2	5	1	3	4	6	9
5	8	9	3	4	6	2	7	1	6	5	8	3	4	9	2	7	6	1	5	8
						1	5	6	8	4	3	9	2	7						
						8	4	3	9	2	7	5	1	6						
						7	9	2	5	6	1	4	3	8						
2	7	5	1	8	3	4	6	9	2	7	5	1	8	3	9	7	4	6	5	2
1	3	6	4	9	7	5	2	8	3	1	9	7	6	4	5	8	2	1	9	3
4	8	9	5	2	6	3	1	7	4	8	6	2	9	5	3	6	1	7	4	8
6	5	2	7	1	9	8	4	3				8	1	6	2	5	9	3	7	4
8	4	3	2	6	5	7	9	1				5	4	2	6	3	7	8	1	9
7	9	1	3	4	8	2	5	6				3	7	9	4	1	8	2	6	5
3	6	4	8	5	1	9	7	2				6	2	1	8	4	5	9	3	7
9	2	8	6	7	4	1	3	5				4	3	8	7	9	6	5	2	1
5	1	7	9	3	2	6	8	4				9	5	7	1	2	3	4	8	6

Hence, total number of solutions = ${}^{100-1}C_{3-1}$

$= {}^{99}C_2 = \frac{99\times 98}{2} = 99\times 49 = 4851$

Thus, the number of ordered triplets = 4851

18. (b): Required number of ways = (Total number of ways in which 5 balls of different colours can be distributed among 3 persons) – (Number of ways in which at least one person gets no ball)

$= 3^5 - ({}^3C_1 \times 2^5 + {}^3C_2 \times 1^5) = 243 - 96 - 3 = 144$

19. (d): We have, ${}^{n-1}C_r = (k^2 - 3)\,{}^nC_{r+1}$

$$\Rightarrow \frac{(n-1)!}{r!(n-1-r)!} = (k^2-3)\frac{n!}{(r+1)!(n-r-1)!}$$

$$\Rightarrow \frac{(n-1)!}{r!(n-1-r)!} = (k^2-3)\frac{n(n-1)!}{(r+1)r!(n-r-1)!}$$

$$\Rightarrow 1 = \frac{(k^2-3)n}{(r+1)}$$

$$\Rightarrow \frac{(r+1)}{n} = k^2-3 \quad [\because\ 0\le r\le n-1 \Rightarrow r+1\le n]$$

$\Rightarrow k^2 - 3 \le 1 \Rightarrow k^2 - 4 \le 0 \Rightarrow -2 \le k \le 2$ ...(i)

Again, ${}^{n-1}C_r = (k^2 - 3)\,{}^nC_{r+1}$

$$\Rightarrow k^2-3 = \frac{{}^{n-1}C_r}{{}^nC_{r+1}} > 0 \Rightarrow k^2 - 3 > 0$$

$\Rightarrow k < -\sqrt{3}$ or $k > \sqrt{3}$ ...(ii)

From (i) and (ii), we have $k \in (\sqrt{3}, 2]$

20. (d): We have, $3^{f(x)} + 2^{-x} = 4 \Rightarrow 3^{f(x)} = 4 - 2^{-x}$

$\Rightarrow f(x) = \log_3(4 - 2^{-x})$.

Clearly, $f(x)$ will be defined, if $4 - 2^{-x} > 0$

$\Rightarrow 2^{-x} < 4 \Rightarrow 2^x > 2^{-2} \Rightarrow x > -2$

$\Rightarrow A = \{x \in R : x > -2\}$.

Now, $f(x) = \log_3(4 - 2^{-x})$

$$\Rightarrow f'(x) = \frac{2^{-x}\log 2}{(4-2^{-x})\log 3} > 0\ \forall\, x > -2$$

$\Rightarrow f(x)$ is increasing on $(-2, \infty)$

$\Rightarrow$ Range of $f(x) = \{y : f(-2) < y < f(\infty)\}$

$\Rightarrow B = \{y : -\infty < y < \log_3 4\} = \{x : -\infty < x < \log_3 4\}$

21. (8): We have, $x^2 - x + 1 = 0$

$$\Rightarrow x = \frac{1\pm\sqrt{1^2-4\cdot1\cdot1}}{2} = \frac{1\pm\sqrt{3}i}{2} = -\omega, -\omega^2$$

For $x = -\omega$, we have

$$\sum_{n=1}^{5}\left(x^n + \frac{1}{x^n}\right)^2 = \sum_{n=1}^{5}\left((-\omega)^n + \frac{1}{(-\omega)^n}\right)^2$$

$$= \sum_{n=1}^{5}(\omega^n + \omega^{2n})^2 = \sum_{\substack{n=1\\ n\neq 3}}^{5}(\omega^n + \omega^{2n})^2 + (\omega^3 + \omega^6)^2$$

$$= \sum_{\substack{n=1\\ n\neq 3}}^{5}(-1)^2 + (2)^2$$

$$\left[\because\ \omega^n + \omega^{2n} = \begin{cases} 2, \text{ if } n \text{ is a multiple of } 3 \\ -1, \text{ if } n \text{ is not a multiple of } 3 \end{cases}\right]$$

= 4 + 4 = 8

Similarly, for $x = -\omega^2$, we obtain that the value of the given expression is 8.

22. (6): We have, $(\sqrt{3}+i)^n = (\sqrt{3}-i)^n$

$$\Rightarrow \left(\frac{\sqrt{3}+i}{\sqrt{3}-i}\right)^n = 1 \Rightarrow \left(\frac{-1+i\sqrt{3}}{1+i\sqrt{3}}\right)^n = 1$$

$$\Rightarrow \left(\frac{2\omega}{-2\omega^2}\right)^n = 1 \left[\because\ \frac{-1+\sqrt{3}i}{2} = \omega \text{ and } \frac{-1-\sqrt{3}i}{2} = \omega^2\right]$$

$\Rightarrow (-1)^n\omega^{2n} = 1$

$\Rightarrow$ n is an even multiple of 3

$\Rightarrow n = 6$

23. (8): We have,

$$f(x) = \begin{vmatrix} 3 & 3x & 3x^2+2a^2 \\ 3x & 3x^2+2a^2 & 3x^3+6a^2x \\ 3x^2+2a^2 & 3x^3+6a^2x & 3x^4+12a^2x^2+2a^4 \end{vmatrix}$$

Applying $C_3 \to C_3 - C_2x$ and $C_2 \to C_2 - C_1x$, we get

$$\begin{vmatrix} 3 & 0 & 2a^2 \\ 3x & 2a^2 & 4a^2x \\ 3x^2+2a^2 & 4a^2x & 6a^2x^2+2a^4 \end{vmatrix}$$

Applying $R_3 \to R_3 - R_2x$ and $R_2 \to R_2 - R_1x$, we get

$$\begin{vmatrix} 3 & 0 & 2a^2 \\ 0 & 2a^2 & 2a^2x \\ 2a^2 & 2a^2x & 2a^2x^2+2a^4 \end{vmatrix} = a^4\begin{vmatrix} 3 & 0 & 2a^2 \\ 0 & 2 & 2x \\ 2 & 2x & 2x^2+2a^2 \end{vmatrix}$$

$= a^4[3(4x^2 + 4a^2 - 4x^2) + 2a^2(-4)] = a^4(12a^2 - 8a^2) = 4a^6$

$\therefore\ f(x) = 4a^6$

$$\int_0^2 4a^6\,dx = 4a^6\left[x\right]_0^2 = 8a^6 = ka^6 \qquad \text{(given)}$$

$\therefore\ k = 8$

24. (4) : $\Delta = \begin{vmatrix} [x]+1 & [y] & [z] \\ [x] & [y]+1 & [z] \\ [x] & [y] & [z]+1 \end{vmatrix}$

$= \begin{vmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ [x] & [y] & [z]+1 \end{vmatrix} \quad \{R_1 \to R_1 - R_2, R_2 \to R_2 - R_3\}$

$= 1([z] + 1 + [y]) + 1([x]) = [x] + [y] + [z] + 1$

$\therefore$ Maximum value is 1 + 0 + 2 + 1 = 4.

25. (8) : We have, $A = \begin{pmatrix} i & -i \\ -i & i \end{pmatrix}$ and $B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$

Now, $A^2 = \begin{pmatrix} i & -i \\ -i & i \end{pmatrix}\begin{pmatrix} i & -i \\ -i & i \end{pmatrix} = \begin{pmatrix} i^2+i^2 & -i^2-i^2 \\ -i^2-i^2 & i^2+i^2 \end{pmatrix}$

$= \begin{pmatrix} 2i^2 & -2i^2 \\ -2i^2 & 2i^2 \end{pmatrix} = \begin{pmatrix} -2 & 2 \\ 2 & -2 \end{pmatrix} = 2\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$

$\therefore \quad A^4 = 2\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \times 2\begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix}$

$= 4\begin{pmatrix} 2 & -2 \\ -2 & 2 \end{pmatrix} = 8\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \Rightarrow A^4 = 8B$

Hence, $k = 8$.

26. (10) : Given that, the system has no unique solution. Therefore either have no solution or infinite solution.

$\therefore \quad |A| = 0$ for both cases.

Now, $\begin{vmatrix} \lambda & -3 & 1 \\ 1 & \lambda & 3 \\ 3 & 1 & 5 \end{vmatrix} = 0 \Rightarrow 5\lambda^2 - 6\lambda - 11 = 0$

$\Rightarrow \quad (5\lambda - 11)(\lambda + 1) = 0 \Rightarrow \lambda = \frac{11}{5}, \lambda = -1$

$\therefore \quad a = 11, b = -1$

$\Rightarrow \quad a + b = 10$

27. (48) : Here, we have 5 digits among which 6 is repeated twice and number greater than 50,000 are consists of 5 digits.

$\therefore$ Number of permutation $= \frac{5!}{2!} = \frac{120}{2} = 60$

But there are some numbers among these 60 numbers which start with 3 and those are less than 50000, so they are to be rejected.

Such number of numbers $= \frac{4!}{2!} = 12$

$\therefore$ The required number of numbers = 60 – 12 = 48

28. (155) : Words before FLOWER are

$1 \times 5! + 1 \times 4! + 1 \times 3! + 2 \times 2! = 154$

$\therefore$ Rank of the word FLOWER is 154 + 1 = 155

29. (96) : The number of words starting with CC = 4!

The number of words starting with CH = 4!

The number of words starting with CI = 4!

The number of words starting with CN = 4!

$\because$ COCHIN is the first word in the list of words beginning with CO.

$\therefore$ Number of words that appear before the word COCHIN = 96

30. (1) : Given $f(x) = \frac{a^x - 1}{x^n(a^x + 1)}$ is even.

$\therefore \quad f(x) = f(-x)$

$$\Rightarrow \frac{a^x - 1}{x^n(a^x + 1)}$$

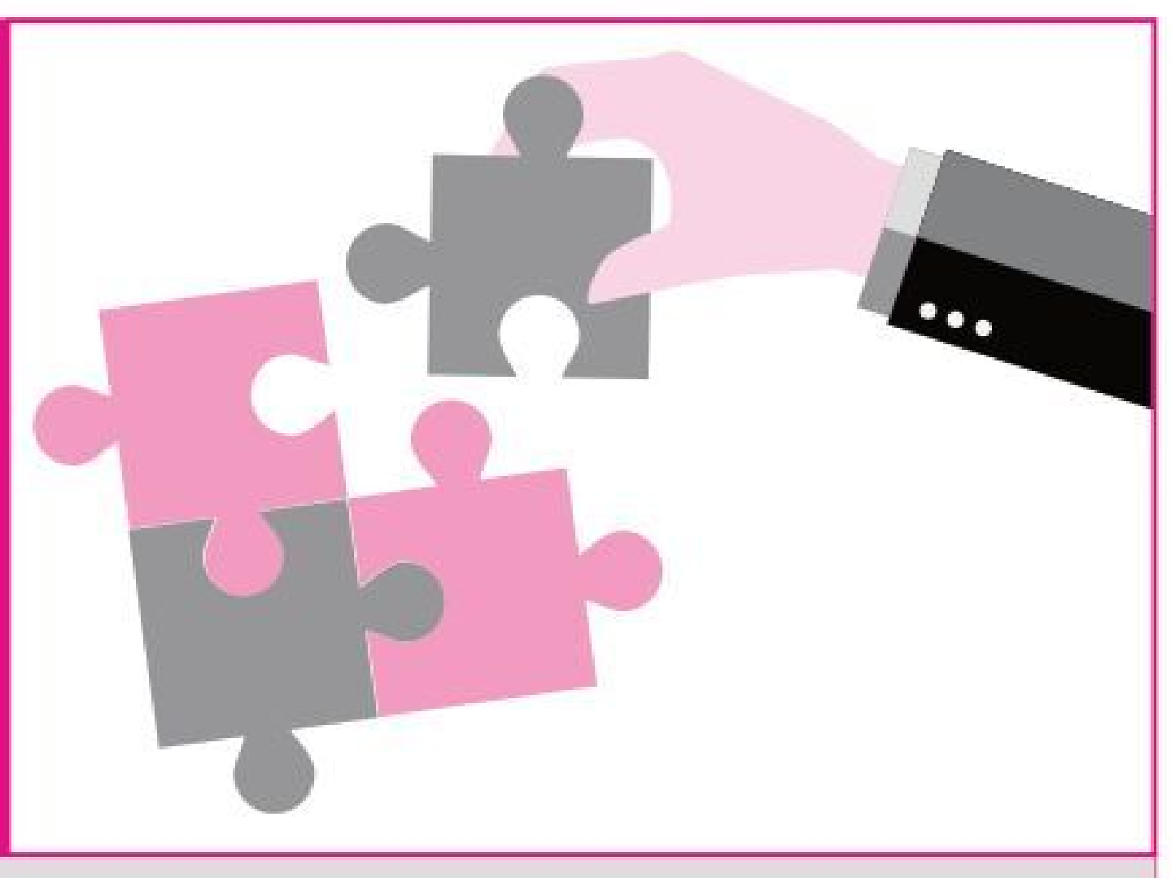
$$= \frac{a^{-x} - 1}{(-x)^n(a^{-x} + 1)} = \frac{1 - a^x}{(-x)^n(1 + a^x)} = \frac{-(a^x - 1)}{(-1)^n x^n (a^x + 1)}$$

$$\Rightarrow 1 = \frac{-1}{(-1)^n}$$

Clearly, the above relation is true for $n = 1$.

◆

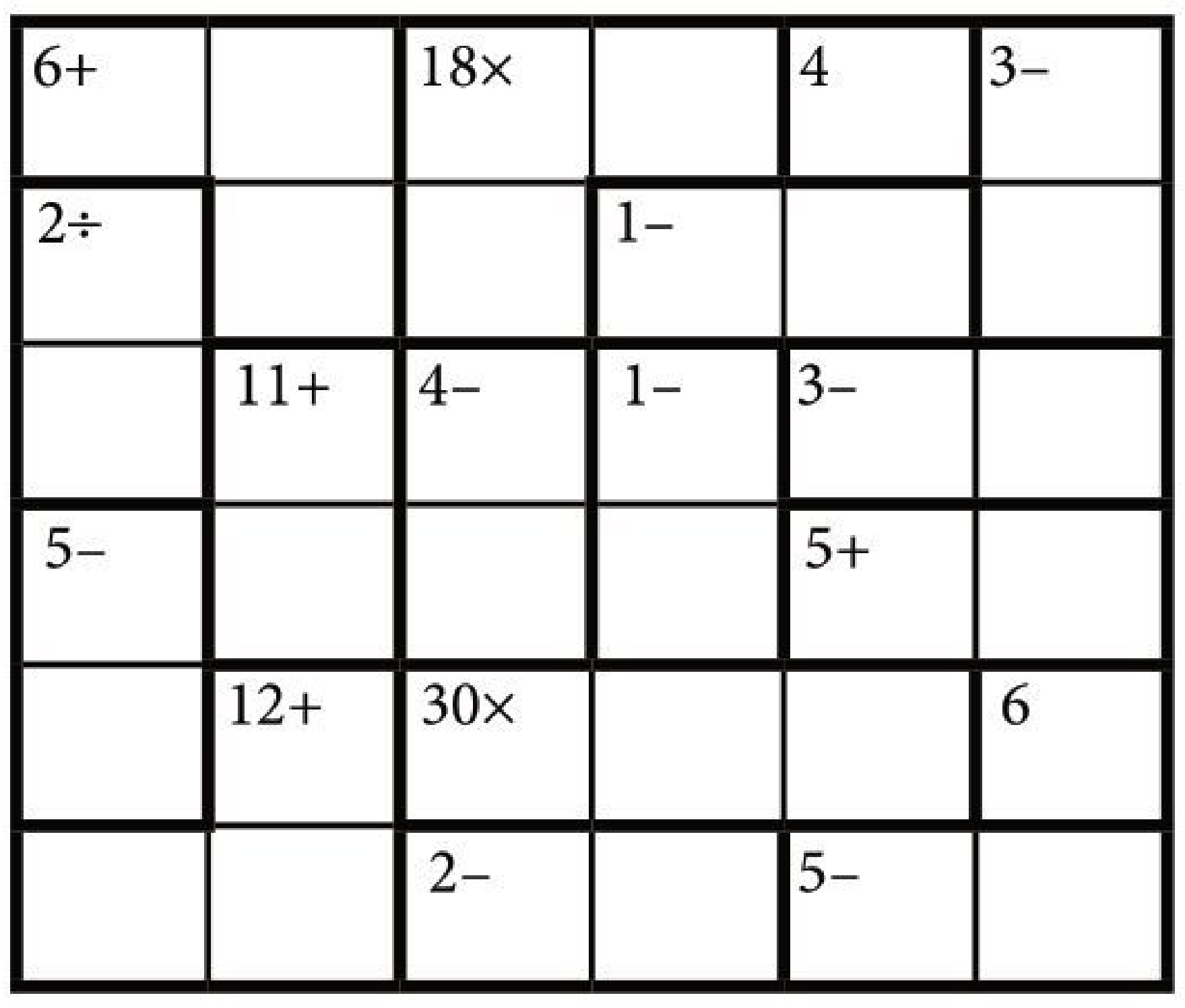
PUZZLE CORNER



MATHDOKU

Introducing MATHDOKU, a mixture of ken-ken, sudoku and Mathematics. In this puzzle 6 × 6 grid is given, your objective is to fill the digits 1-6 so that each appear exactly once in each row and each column.

Notice that most boxes are part of a cluster. In the upper-left corner of each multibox cluster is a value that is combined using a specified operation on its numbers. For example, if that value is 3 for a two-box cluster and operation is multiply, you know that only 1 and 3 can go in there. But it is your job to determine which number goes where! A few cluster may have just one box and that is the number that fills that box.



6+		18×		4	3–
2÷			1–		
	11+	4–	1–	3–	
5–				5+	
	12+	30×			6
		2–		5–	

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

beat the
TIME TRAP

Duration : 30 minutes

SECTION-I

Single Option Correct Type

1. Equation of the tangent to the circle, at the point (1, –1), whose centre is the point of intersection of the straight lines $x - y = 1$ and $2x + y = 3$ is

(a) $x + 4y + 3 = 0$ (b) $3x - y - 4 = 0$
(c) $x - 3y - 4 = 0$ (d) $4x + y - 3 = 0$

2. The equation of a normal to the curve,

$\sin y = x\sin\left(\frac{\pi}{3}+y\right)$ at $x = 0$, is

(a) $2x+\sqrt{3}y=0$ (b) $2y-\sqrt{3}x=0$
(c) $2y+\sqrt{3}x=0$ (d) $2x-\sqrt{3}y=0$

3. If for a positive integer n, the quadratic equation $x(x+1)+(x+1)(x+2)+......+(x+\overline{n-1})(x+n)=10n$ has two consecutive integral solutions, then n is equal to

(a) 9 (b) 10 (c) 11 (d) 12

4. If $\tan\theta + \tan2\theta + \tan3\theta = \tan\theta \tan2\theta \tan3\theta$, then the general value of θ is

(a) $n\pi, n \in Z$ (b) $\frac{n\pi}{6}, n \in Z$
(c) $n\pi \pm \frac{\pi}{3}, n \in Z$ (d) $\frac{n\pi}{2}, n \in Z$

5. If the four faces of a tetrahedron are represented by the equations $\vec{r}\cdot(\beta\hat{i}+\gamma\hat{j})=0, \vec{r}\cdot(\gamma\hat{j}+\alpha\hat{k})=0$, $\vec{r}\cdot(\alpha\hat{k}+\beta\hat{i})=0$ and $\vec{r}\cdot(\beta\hat{i}+\gamma\hat{j}+\alpha\hat{k})=\lambda$, then volume of tetrahedron (in cubic units)
(where $\beta, \gamma, \alpha, \lambda$ are positive numbers) is

(a) $\frac{1}{6}\frac{\lambda^3}{(\alpha\beta y)^3}$ (b) $\frac{\lambda^3}{\alpha\beta y}$
(c) $\frac{\lambda^3}{3\alpha\beta y}$ (d) $\frac{\lambda^3}{6\alpha\beta\gamma}$

6. Let $\vec{n}$ be a vector of magnitude $2\sqrt{3}$ such that it makes acute equal angles with the co-ordinate axes, the vector form of equation of plane passing through (1, –1, 2) and normal to $\vec{n}$ is

(a) $\vec{r}\cdot(\hat{i}+2\hat{j}+3\hat{k})=2$ (b) $\vec{r}\cdot(2\hat{i}-3\hat{j}+\hat{k})=2$
(c) $\vec{r}\cdot(2\hat{i}+\hat{j}+\hat{k})=2$ (d) $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$

7. If (4, 0) and (–4, 0) be the vertices and (6, 0) and (–6, 0) be the foci of a hyperbola, then its eccentricity is

(a) 5/2 (b) 2 (c) 3/2 (d) $\sqrt{2}$

8. If $[x]$ stands for the greatest integer function, then the value of $\int_4^{10}\frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$ is

(a) 0 (b) 1
(c) 3 (d) none of these

9. If $f(n)=\sum_{k=1}^{n}\sum_{j=k}^{n}({}^nC_j)({}^jC_k)$, then the value of $f(6)$ is

(a) 660 (b) 665
(c) 670 (d) none of these

10. $\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\frac{4}{5}x^5+......\text{ to }\infty=$

(a) $\frac{x}{1-x}+\log(1-x)$ (b) $\frac{x}{1+x}+\log(1+x)$
(c) $-\frac{x}{1+x}+\log(1+x)$ (d) none of these

SECTION-II

Numerical Answer Type

11. The distance from the point $-\hat{i}+2\hat{j}+6\hat{k}$ to the straight line passing through the point with position vector $2\hat{i}+3\hat{j}-4\hat{k}$ and parallel to the vector $6\hat{i}+3\hat{j}-4\hat{k}$ is ______.

12. If $\int \operatorname{cosec}\left(x-\frac{\pi}{6}\right)\operatorname{cosec}\left(x-\frac{\pi}{3}\right)dx = a\ln\left(\frac{\sin\left(x-\frac{\pi}{3}\right)}{\sin\left(x-\frac{\pi}{6}\right)}\right)+b,$

then a is equal to ______.

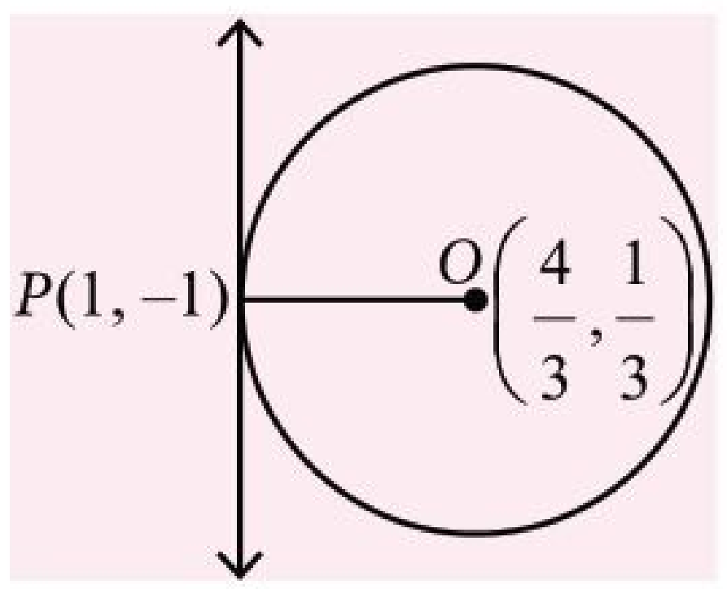
13. A coffee connoisseur claims that he can distinguish between a cup of instant coffee and a cup of percolator coffee 75% of time. It is agreed that his claim will be accepted if he correctly identifies at least 5 of the 6 cups. His chances of having the claim accepted is ______.

14. Normals AO, AA_1, AA_2 are drawn to parabola $y^2 = 8x$ from the point $A(h, 0)$. If triangle OA_1A_2 is equilateral, then possible value of h is ______.

15. If $|x - y| + |x + y| = 2$, then max. possible value of $x^2 - 6x + y^2$ is ______.

SOLUTIONS

1. (a) : Point of intersection of lines $x - y = 1$ and $2x + y = 3$ is $O\left(\frac{4}{3}, \frac{1}{3}\right)$.



Slope of $OP = \frac{\frac{1}{3}+1}{\frac{4}{3}-1} = \frac{\frac{4}{3}}{\frac{1}{3}} = 4$

$\therefore$ Slope of tangent $= -\frac{1}{4}$

So, equation of tangent at $P(1, -1)$ is

$y+1 = -\frac{1}{4}(x-1) \Rightarrow x + 4y + 3 = 0$

2. (a) : We have, $\sin y = x\sin\left(\frac{\pi}{3}+y\right)$ ...(i)

Differentiating (i) w.r.t. x, we get

$\cos y\frac{dy}{dx} = x\cos\left(\frac{\pi}{3}+y\right)\frac{dy}{dx}+\sin\left(\frac{\pi}{3}+y\right)$ ...(ii)

Putting $x = 0$ and $y = 0$ in (ii), we get

$\frac{dy}{dx} = \frac{\sqrt{3}}{2} \Rightarrow \frac{-dx}{dy} = \frac{-2}{\sqrt{3}}$

$\therefore$ Equation of normal passing through (0, 0) is $y = \frac{-2}{\sqrt{3}}x$

$\Rightarrow \sqrt{3}y = -2x \Rightarrow 2x+\sqrt{3}y = 0$

3. (c) : We have, $\sum_{k=1}^{n}(x+k-1)(x+k) = 10n$

$\Rightarrow \sum_{k=1}^{n}[x^2+(2k-1)x+k(k-1)] = 10n$

$\Rightarrow nx^2+n^2x+\frac{1}{3}n(n^2-1) = 10n$

$\Rightarrow x^2+nx+\frac{1}{3}(n^2-1)-10 = 0$

$\Rightarrow 3x^2 + 3nx + n^2 - 31 = 0$

Let consecutive roots be m and $m + 1$, then

$(m + 1 - m)^2 = (m + 1 + m)^2 - 4m(m + 1)$

$\Rightarrow 1 = n^2 - 4\left(\frac{n^2-31}{3}\right) \Rightarrow n^2 = 121 \therefore n = 11$ [$\because n > 0$]

4. (b) : Given, $\tan\theta + \tan2\theta + \tan3\theta = \tan\theta\tan2\theta\tan3\theta$ ...(i)

We know that

$\tan6\theta = \frac{\tan\theta+\tan2\theta+\tan3\theta-\tan\theta\tan2\theta\tan3\theta}{1-\sum\tan\theta\tan2\theta}$

$= 0$ (Using (i))

$\Rightarrow 6\theta = n\pi \Rightarrow \theta = n\pi/6, n \in Z$

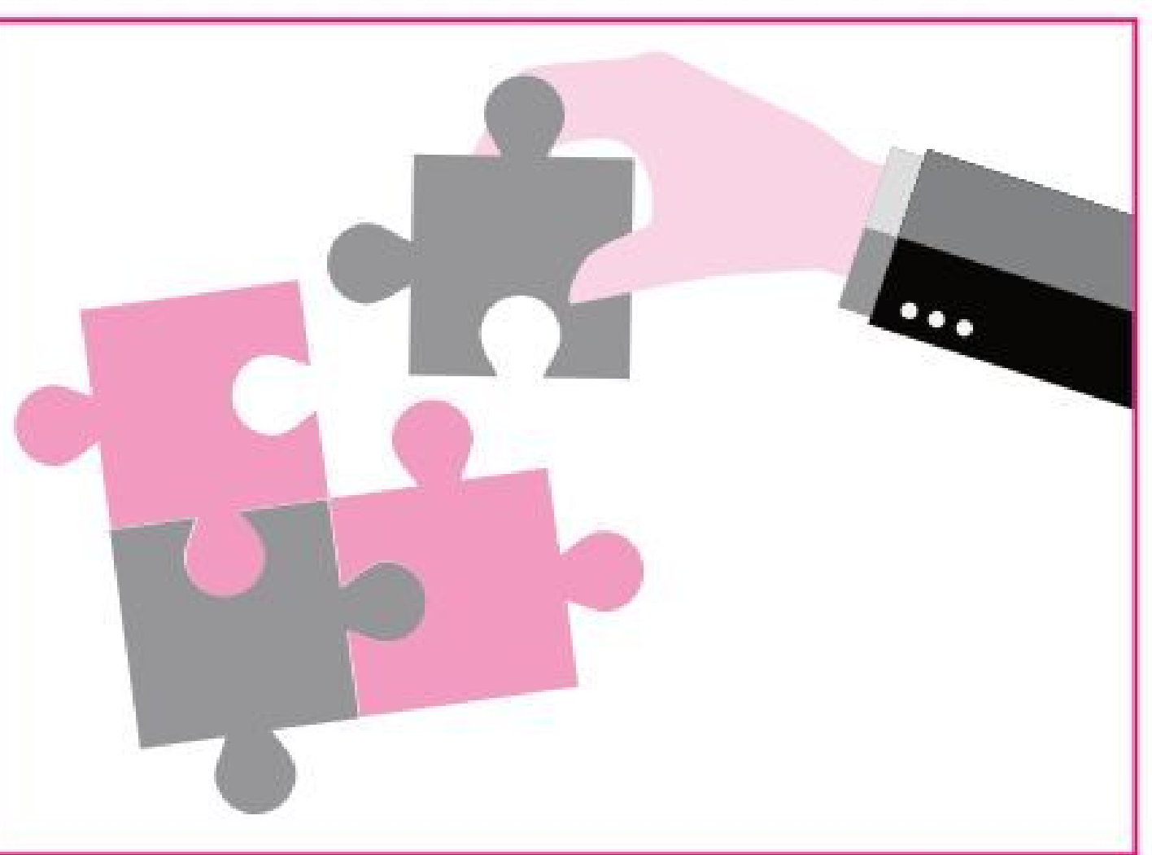
5. (d) : Cartesian equations of the planes are

$\beta x + \gamma y = 0$ ...(i) $\gamma y + \alpha z = 0$ ...(ii)

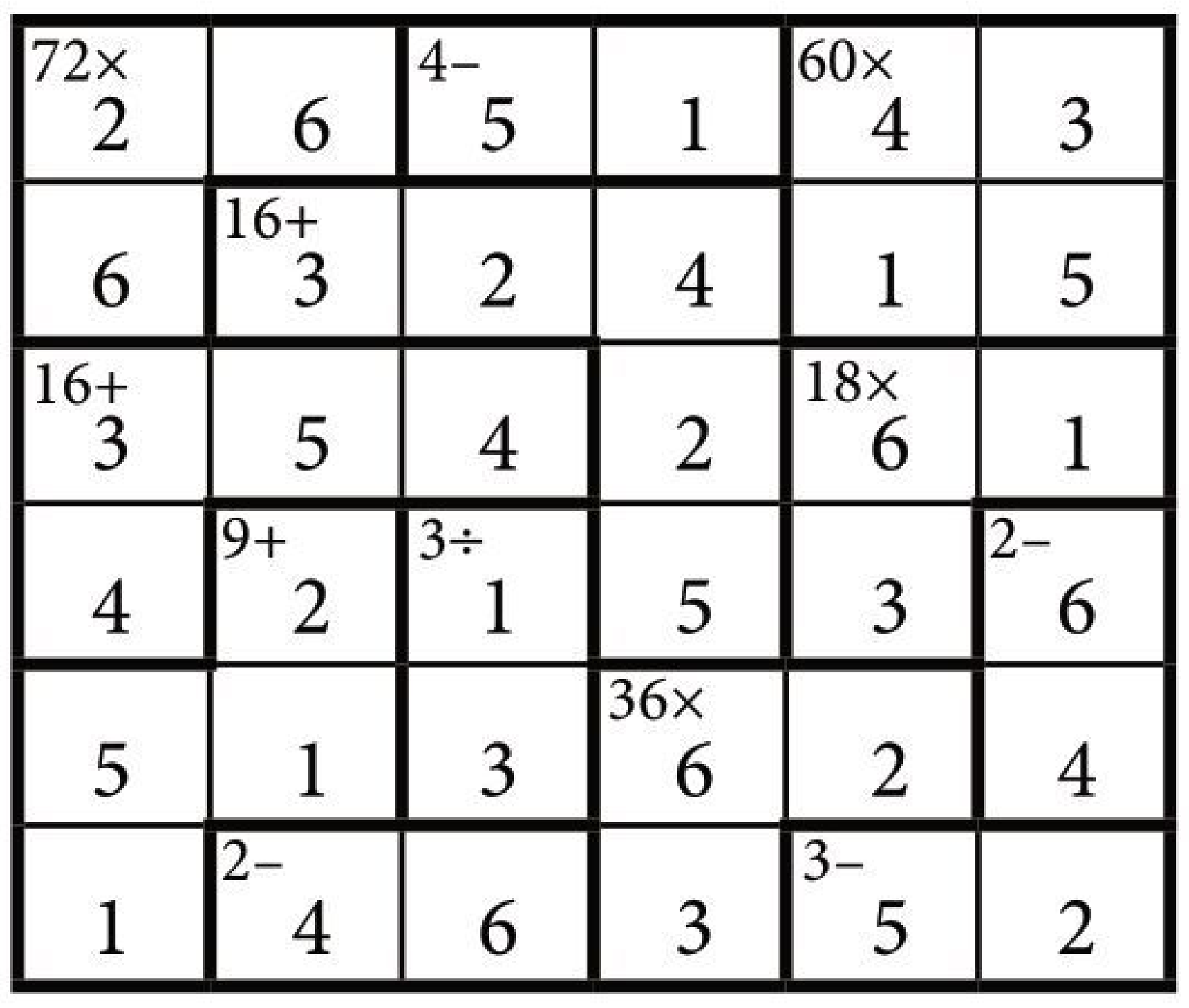
$\alpha z + \beta x = 0$ ...(iii) and $\beta x + \gamma y + \alpha z = \lambda$ ...(iv)

Now, plane (iv) cuts the plane (i) at $\left(0, 0, \frac{\lambda}{\alpha}\right)$, the plane (iv) cuts the plane (ii) at $\left(\frac{\lambda}{\beta}, 0, 0\right)$ and the plane (iv) cuts the plane (iii) at $\left(0, \frac{\lambda}{\gamma}, 0\right)$. Thus the planes (i), (ii) and (iii) are coordinate planes intersecting at origin (0, 0, 0). Therefore the lengths of the coterminous edges of the tetrahedron are $\left(\frac{\lambda}{\beta}, \frac{\lambda}{\gamma}, \frac{\lambda}{\alpha}\right)$

PUZZLE CORNER



ANSWER - SEPTEMBER 2021



72× 2	6	4– 5	1	60× 4	3
6	16+ 3	2	4	1	5
16+ 3	5	4	2	18× 6	1
4	9+ 2	3÷ 1	5	3	2– 6
5	1	3	36× 6	2	4
1	2– 4	6	3	3– 5	2

$\therefore$ Volume of tetrahedron $=\frac{1}{6}\left(\frac{\lambda}{\beta}\cdot\frac{\lambda}{\gamma}\cdot\frac{\lambda}{\alpha}\right)=\frac{\lambda^3}{6\alpha\beta\gamma}$

6. (d) : Given that $\alpha=\beta=\gamma \Rightarrow \cos\alpha=\cos\beta=\cos\gamma$

i.e., $l=m=n \quad \therefore \; l=\frac{1}{\sqrt{3}} \quad (\because l^2+m^2+n^2=1)$

$$\therefore \; \vec{n}=2\sqrt{3}\left(\frac{\hat{i}+\hat{j}+\hat{k}}{\sqrt{3}}\right) \qquad [\because \vec{r}=|\vec{r}|(l\hat{i}+m\hat{j}+n\hat{k})]$$

Thus the required vector equation of plane is

$\vec{r}\cdot(2\hat{i}+2\hat{j}+2\hat{k})=(\hat{i}-\hat{j}+2\hat{k})(2\hat{i}+2\hat{j}+2\hat{k})$

$\Rightarrow \vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=2$

7. (c) : Vertices : $(\pm 4, 0)\equiv(\pm a, 0)\Rightarrow a=4$

Foci : $(\pm 6, 0)\equiv(\pm ae, 0) \Rightarrow e=\frac{6}{4}=\frac{3}{2}$

8. (c) : Let $I=\int_4^{10}\frac{[x^2]dx}{[x^2-28x+196]+[x^2]}$

$$\Rightarrow I=\int_4^{10}\frac{[x^2]dx}{[(14-x)^2]+[x^2]} \qquad \text{...(i)}$$

$$=\int_4^{10}\frac{\left[(14-x)^2\right]dx}{[x^2]+\left[(14-x)^2\right]} \qquad \text{...(ii)}$$

$$\left[\because \int_a^b f(x)dx=\int_a^b f(a+b-x)dx\right]$$

Adding (i) and (ii), we get

$$2I=\int_4^{10}1dx=6 \Rightarrow I=3$$

9. (b) : $f(n)=\sum_{j=1}^{n}({}^nC_j)({}^jC_1)+\sum_{j=2}^{n}({}^nC_j)({}^jC_2)+....$

$$+\sum_{j=n-1}^{n}({}^nC_j)({}^jC_{n-1})+({}^nC_n)({}^nC_n)$$

$=({}^nC_1)({}^1C_1)+({}^nC_2)({}^2C_1+{}^2C_2)+({}^nC_3)({}^3C_1+{}^3C_2+{}^3C_3)$
$+ ... + ({}^nC_n)({}^nC_1+{}^nC_2+ ... +{}^nC_n)$

$=({}^nC_1)(2-1)+({}^nC_2)(2^2-1)+({}^nC_3)(2^3-1)+ ...$
$+{}^nC_n(2^n-1)$

$=\{(2+1)^n-1\}-\{(1+1)^n-1\}=3^n-2^n$

$\therefore f(6)=3^6-2^6=665$

10. (a) : The given series is

$\frac{1}{2}x^2+\frac{2}{3}x^3+\frac{3}{4}x^4+\frac{4}{5}x^5+.......\text{ to }\infty$

$=\frac{2-1}{2}x^2+\frac{3-1}{3}x^3+\frac{4-1}{4}x^4+\frac{5-1}{5}x^5+.......\text{ to }\infty$

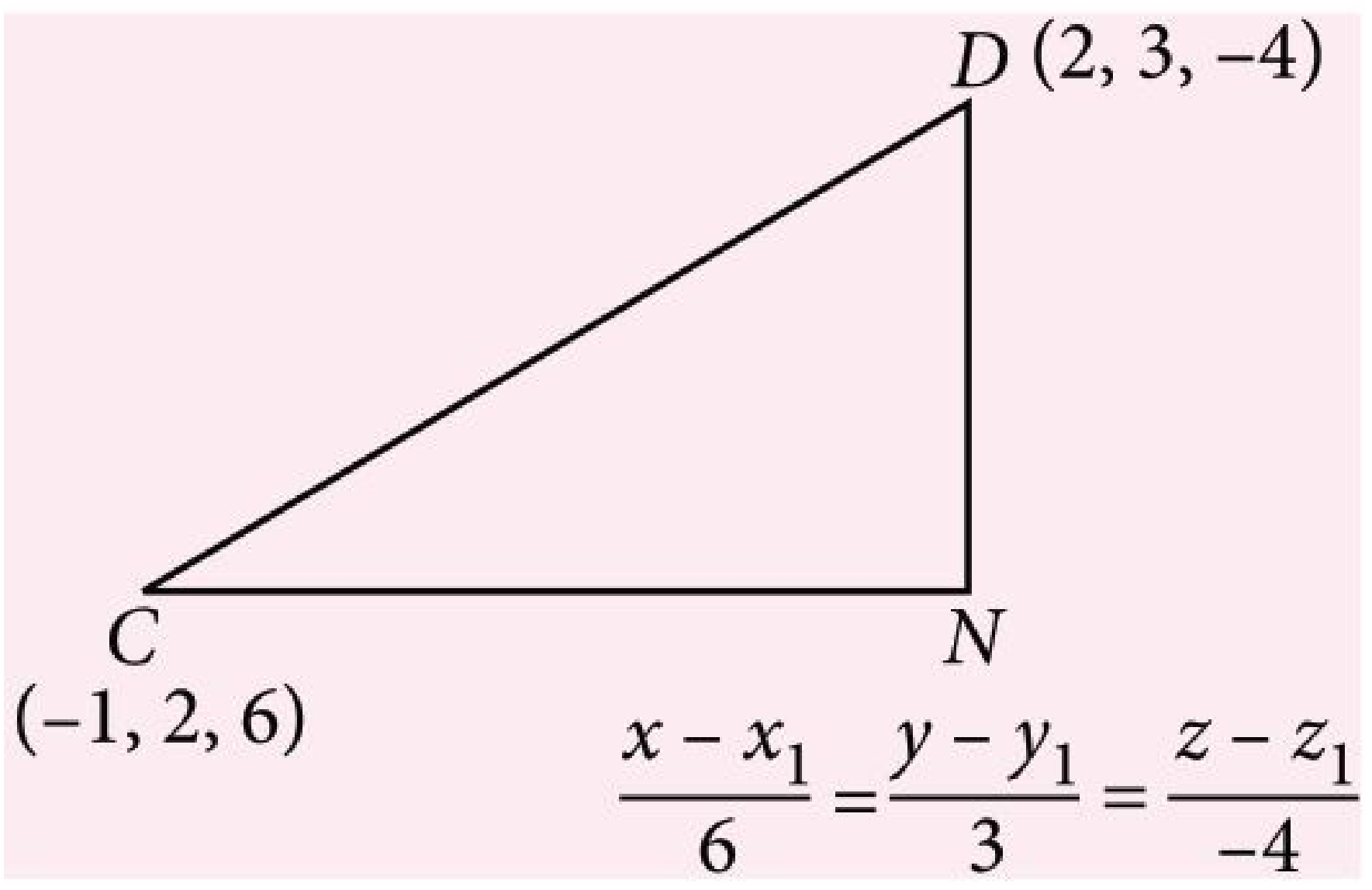
$=(x^2+x^3+x^4+x^5+....\text{ to }\infty)-\left(\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+....\text{ to }\infty\right)$

$=\frac{x^2}{1-x}+x+\log(1-x)=\frac{x^2+x-x^2}{1-x}+\log(1-x)$

$=\frac{x}{1-x}+\log(1-x)$

11. (7) : Let C is a point with position vector $-\hat{i}+2\hat{j}+6\hat{k}$ and D is a point with position vector $2\hat{i}+3\hat{j}-4\hat{k}$

$\therefore \; CD=3\hat{i}+\hat{j}-10\hat{k}$



DN = Projection of CD on DN

$= l(x_2-x_1)+m(y_2-y_1)+n(z_2-z_1)$

where l, m and n are $\frac{6}{\sqrt{61}}, \frac{3}{\sqrt{61}}, \frac{-4}{\sqrt{61}}$

$=\frac{6}{\sqrt{61}}\times 3+\frac{3}{\sqrt{61}}\times 1+\frac{(-4)(-10)}{\sqrt{61}}=\sqrt{61}$

$\therefore \; CN=\sqrt{(CD)^2-(DN)^2}=\sqrt{3^2+1^2+(-10)^2-61}=7$

12. (2) : Let $I=\int \operatorname{cosec}\left(x-\frac{\pi}{6}\right)\operatorname{cosec}\left(x-\frac{\pi}{3}\right)dx$

$$=\int\frac{2\sin\frac{\pi}{6}}{\sin\left(x-\frac{\pi}{6}\right)\sin\left(x-\frac{\pi}{3}\right)}dx$$

$$=2\int\frac{\sin\left\{\left(x-\frac{\pi}{6}\right)-\left(x-\frac{\pi}{3}\right)\right\}}{\sin\left(x-\frac{\pi}{6}\right)\sin\left(x-\frac{\pi}{3}\right)}dx$$

MONTHLY TEST DRIVE CLASS XII ANSWER KEY

1.	(b)	2.	(c)	3.	(b)	4.	(c)	5.	(d)
6.	(b)	7.	(c,d)	8.	(b,d)	9.	(d)		
10.	(a,b,c)	11.	(a,b)	12.	(a,c)	13.	(b,c)	14.	(a)
15.	(b)	16.	(c)	17.	(3)	18.	(0)	19.	(15)
20.	(2)								

$$=2\int\left[\cot\left(x-\frac{\pi}{3}\right)-\cot\left(x-\frac{\pi}{6}\right)\right]dx$$

$$=2\ln\left\{\frac{\sin\left(x-\frac{\pi}{3}\right)}{\sin\left(x-\frac{\pi}{6}\right)}\right\}+b$$

13. (0.53) : Let p denotes the probability of a correct distinction between a cup of instant coffee and a cup of percolator coffee.

Then, $p=\frac{75}{100}=\frac{3}{4} \Rightarrow q=1-p=\frac{1}{4}$ and $n=6$

Let random variable X denotes the number of correct distinctions.

Then, $P(X=x)=P(x)={}^6C_x\left(\frac{3}{4}\right)^x\left(\frac{1}{4}\right)^{6-x}$, $x=0,1,\ldots,6$

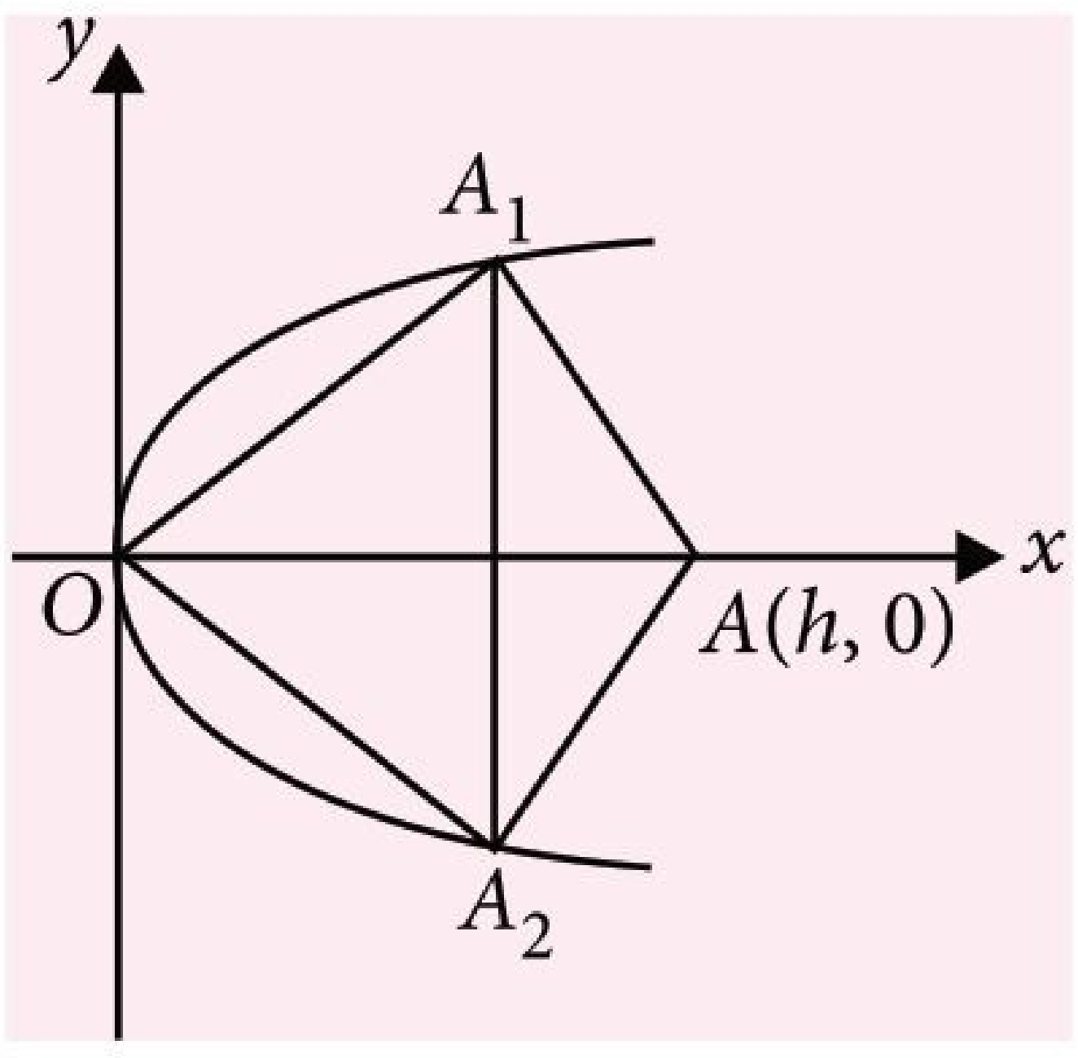
The probability of the claim being accepted is

$$P(X\ge 5)=P(5)+P(6)={}^6C_5\left(\frac{3}{4}\right)^5\left(\frac{1}{4}\right)^{6-5}+{}^6C_6\left(\frac{3}{4}\right)^6$$

$$=6\times\frac{3^5}{4^6}+1\times\frac{3^6}{4^6}=\frac{3^6}{4^6}(2+1)=\frac{3^7}{4^6}$$

= 0.53 (approx.)

14. (28) : Let $A_1 \equiv (2t_1^2, 4t_1)$,
$A_2 \equiv (2t_1^2, -4t_1)$



Slope of $OA_1 = \frac{4t_1}{2t_1^2}=\frac{2}{t_1}$

Clearly, $\angle A_1OA = \pi/6$

[$\because$ OA_1A_2 is equilateral triangle]

$\therefore \frac{2}{t_1}=\frac{1}{\sqrt{3}} \Rightarrow t_1=2\sqrt{3}$

Equation of normal at A_1 is $y=-t_1x+4t_1+2t_1^3$

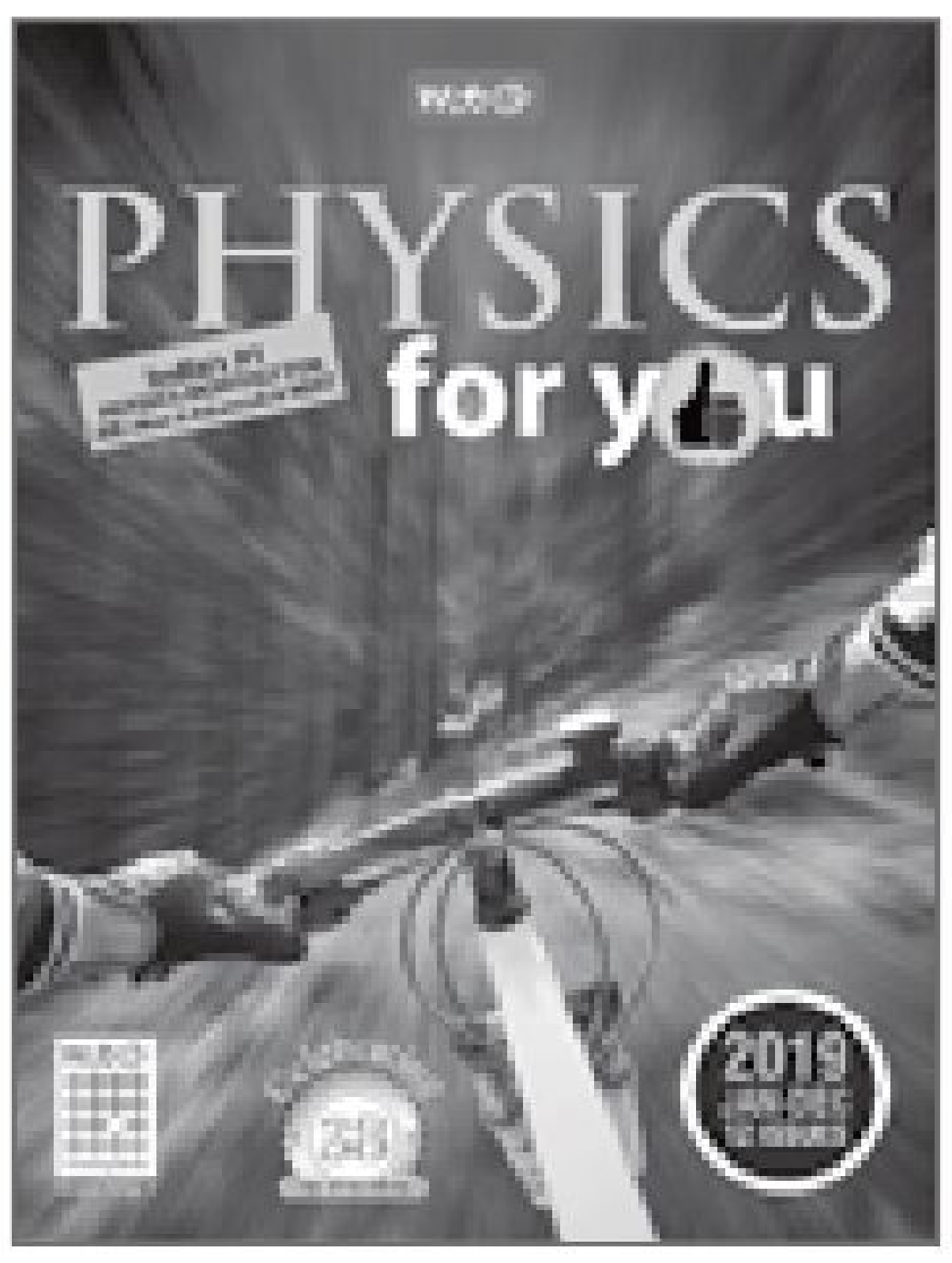
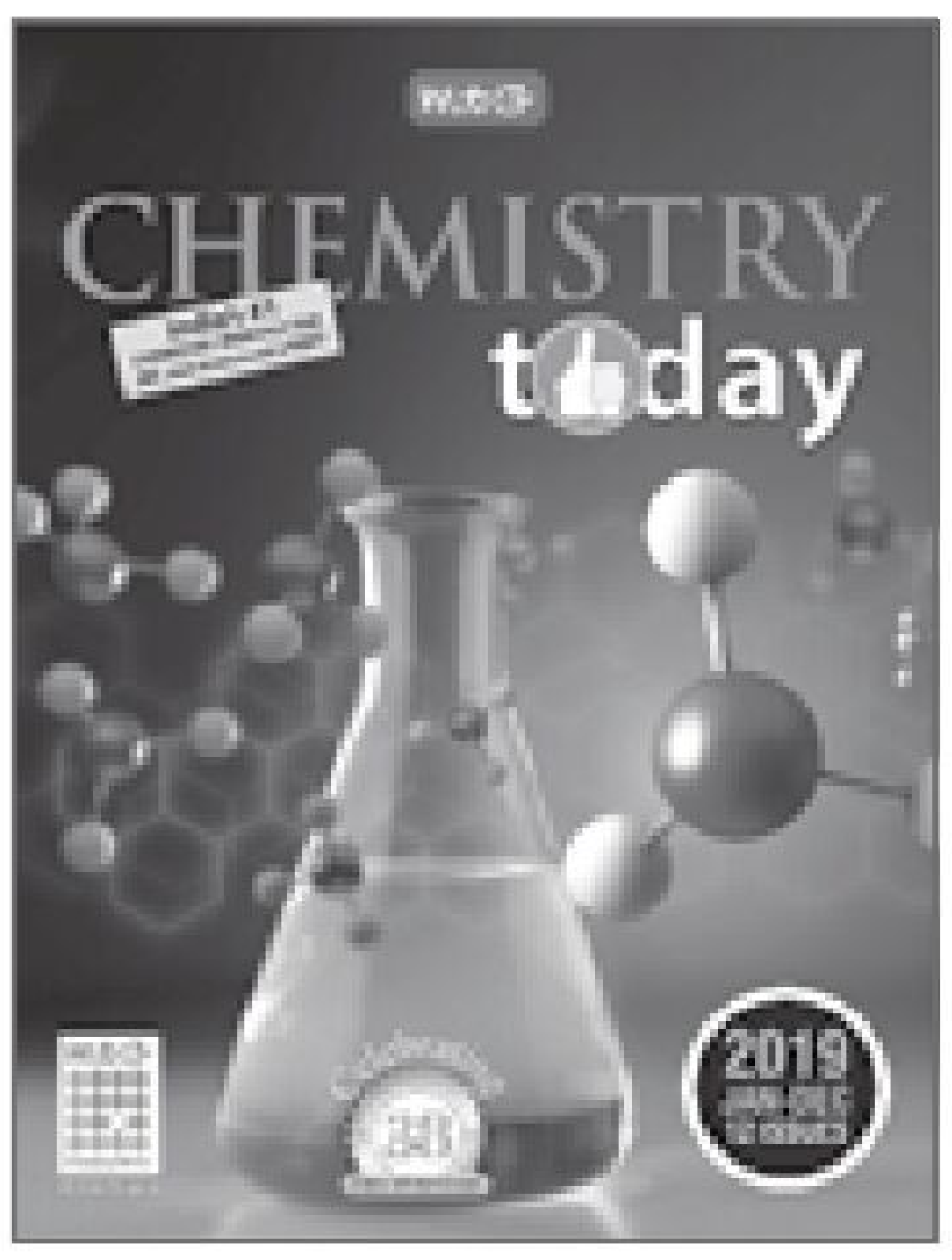
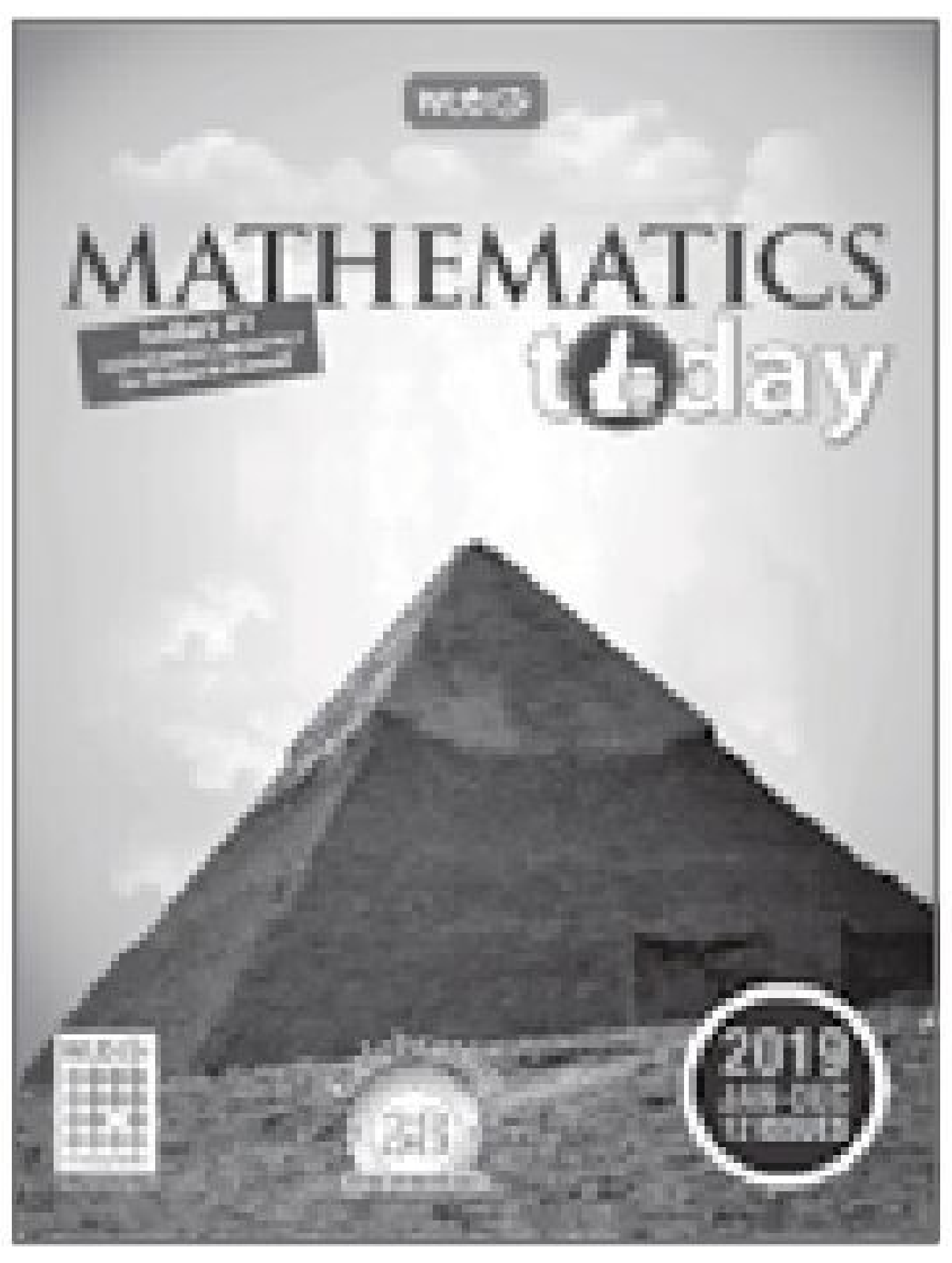
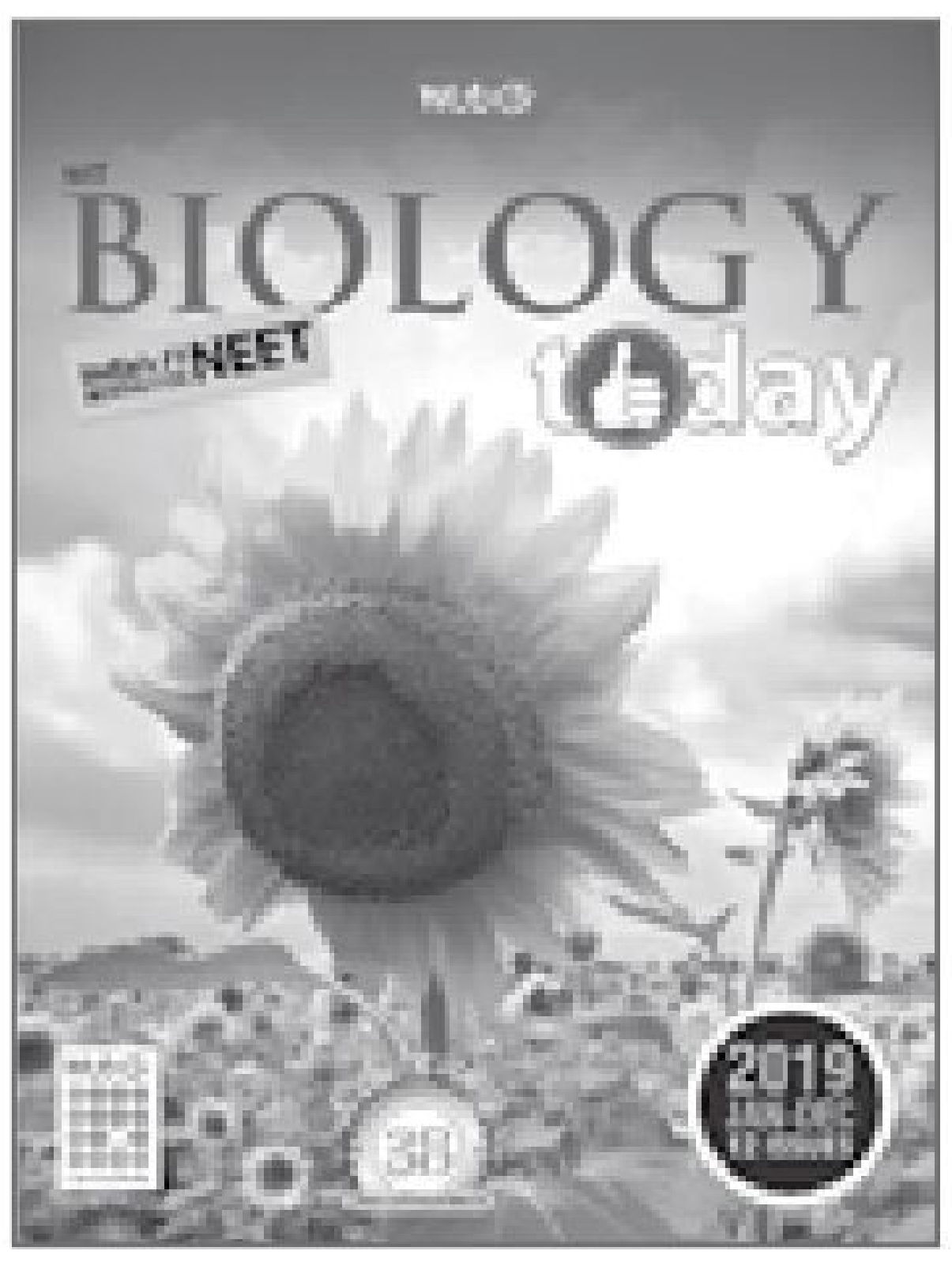
$\Rightarrow h=4+2t_1^2$ [$\because$ It passes through $(h, 0)$]

$= 4 + 2(12) = 28$

15. (8) : $|x-y|+|x+y|=2$ represents a square with vertices at $(\pm1, \pm1)$ and $x^2-6x+y^2=(x-3)^2+y^2-9$ is the distance of any point on the square from $(3, 0)$. Thus, maximum possible value is given by

$(-1-3)^2+(1)^2-9=16+1-9=8$

◆ ◆

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OLYMPIAD CORNER

1. An exam paper consists of 5 multiple-choice questions, each with 4 different choices; 2000 students take the test, and each student chooses exactly one answer per question. Find the smallest value of n for which it is possible for the students' answer sheets to have the following property: among any n of the students' answer sheets, there exist 4 of them among which any two have at most 3 common answers.

2. Let m, n be positive integers such that $m \le \frac{n^2}{4}$ and every prime divisor of m is less than or equal to n. Show that m divides $n!$.

3. If $g(x)$ and $h(x)$ are polynomials with real coefficients and $f(x) = g(x^3) + xh(x^3)$ is divisible by $x^2 + x + 1$, show that $g(x)$ and $h(x)$ are both divisible by $x - 1$.

4. If $A_1, A_2, \ldots, A_n$ is an n - sided regular polygon such that $\frac{1}{A_1A_2} = \frac{1}{A_1A_3} + \frac{1}{A_1A_4}$, then find n, the number of sides of the polygon.

5. Does there exist a bijection f of

(a) a plane with itself

(b) three-dimensional space with itself such that for any distinct points A, B line AB and line $f(A)\,f(B)$ are perpendicular?

6. For which acute-angled triangle is the ratio of the shortest side to the inradius maximal?

7. Determine the maximum value of λ such that if $f(x) = x^3 + ax^2 + bx + c$ is a cubic polynomial with all its roots non-negative, then $f(x) \ge \lambda(x - a)^3$ for all $x \ge 0$. Find the equality condition.

SOLUTIONS

1. First we prove that $n \ge 25$. Let 1, 2, 3, 4 denote the four different choices of each problem. Represent each student's answer sheet by an ordered 5-tuple $(a_1, a_2, a_3, a_4, a_5)$, $a_i \in \{1, 2, 3, 4\}$, where the student's answer to problem i is a_i. We say that two answer sheets are of the same type if their corresponding 5-tuples belong to a set of the form

$$\{(k, a_2, a_3, a_4, a_5) | k \in \{1, 2, 3, 4\}\},$$

where $a_2, a_3, a_4, a_5 \in \{1, 2, 3, 4)\}$. Since there are 256 such sets and $2000 = 256 \times 7 + 208$, at least eight answer sheets are of the same type by the pigeonhole Principle. Among the 1992 remaining answer sheets, again some eight are of the same type. Finally, among the 1984 remaining answer sheets, another eight are of the same type. Consider the set A of these 24 answer sheets. Given any four answer sheets in A, two of them must be of the same type, that is, their solutions for the last 4 problems are identical. This violates the assumption that there are 4 answer sheets in A, among which any two have at most 3 common answers. Hence, $n \ge 25$.

Now we show that $n = 25$ is indeed attainable. Define the set $S = \{(a_1, a_2, a_3, a_4, a_5) | \sum_{i=1}^{5} a_i \equiv 0 \pmod 4),\ a_i \in \{1, 2, 3, 4\}\}$.

Then $|S| = 4^4 = 256$, and any two answer sheets have at most 3 common answers if their corresponding 5-tuples are distinct elements of S. Pick any 250 elements of S and assume that exactly eight students turn in answer sheets that correspond to each of these 250 5-tuples. Among any $25 > 3.8$ answer sheets, there are four whose corresponding 5-tuples are distinct elements in S, and they satisfy the given conditions of the problem.

Therefore, the answer is $n = 25$.

2. It suffices to show that $p^kn!$ for all primes p and integers $k \ge 1$ such that p^km. If $k = 1$, we are done because we are given that $p \le n$ and hence $p|n!$. Otherwise, because $m \le \frac{n^2}{4}$, we have $p^k \le \frac{n^2}{4}$. Thus, $n \ge 2\sqrt{p^k}$. If $n \ge kp$, then at least k of the numbers 1, 2, ..., n are multiples of p, implying that $p^k|n!$. Hence, it suffices to show that $2\frac{\sqrt{p^k}}{p} \ge k$ or equivalently that

$$p^{\frac{(k-2)}{2}} \ge \frac{k}{2}, \qquad (*)$$

because this implies $n \ge 2\sqrt{p^k} \ge kp$.

Indeed, we can do so for most values p and k. If $k = 2$, then (*) reads $1 \ge 1$. If $k \ge 4$, then Bernoulli's inequality implies that $p^{\frac{(k-2)}{2}} = (1+(p-1))^{\frac{(k-2)}{2}} \ge 1+\frac{k-2}{2}(p-1) \ge \frac{k}{2}$, as desired. Finally, if $k = 3$, then (*) is clearly true unless $p = 2$. If indeed $k = 3$ and $p = 2$, then $m \ge 8$, $n \ge 6$ and $n!$ is certainly divisible by 8. Thus, we have shown that $p^k | n!$ in all cases, completing the proof.

3. **1st solution:** If $f(x) = g(x^3) + xh(x^3)$ is divisible by $x^2 + x + 1$, we can write $f(x) = q(x)(x^2 + x + 1)$. Let ω be a non - real cube root of unity. Then $f(\omega) = q(\omega)(\omega^2 + \omega + 1) = 0$ and $f(\omega^2) = q(\omega^2)(\omega + \omega^2 + 1) = 0$. Therefore $g(1) + \omega h(1) = 0$ and $q(1) + \omega^2 h(1) = 0$. So, it follows that $g(1) = h(1) = 0$; therefore both $g(x)$ and $h(x)$ are divisible by $x - 1$.

2nd solution: we can write $g(x) = (x - 1)g'(x) + g(1)$.
Then, $g(x^3) = (x^3 - 1)\, g'(x^3) + g(1)$. Similarly, we can write $h(x^3) = (x^3 - 1)\, h'(x^3) + h(1)$. Therefore, $f(x) = (x^3 - 1) g'(x^3) + xh'(x^3)) + g(1) + xh(1)$.
Since $x^2 + x + 1$ divides both $f(x)$ and $x^3 - 1$, it also divides $g(1) + xh(1)$. Therefore $g(1) = h(1) = 0$.

4. We can assume that the radius of the circumcircle of the polygon is 1. Let 2θ be the angle subtended by any side of the polygon at the center of the circumcircle. We know, when a segment of the circle subtends an angle ϕ at the center, its length is $2\sin\frac{\phi}{2}$. Therefore, $A_1A_2 = 2\sin\theta$, $A_1A_3 = 2\sin 2\theta$ and $A_1A_4 = 2\sin 3\theta$. Now by hypothesis,

$$\frac{1}{\sin\theta} = \frac{1}{\sin 2\theta} + \frac{1}{\sin 3\theta}$$

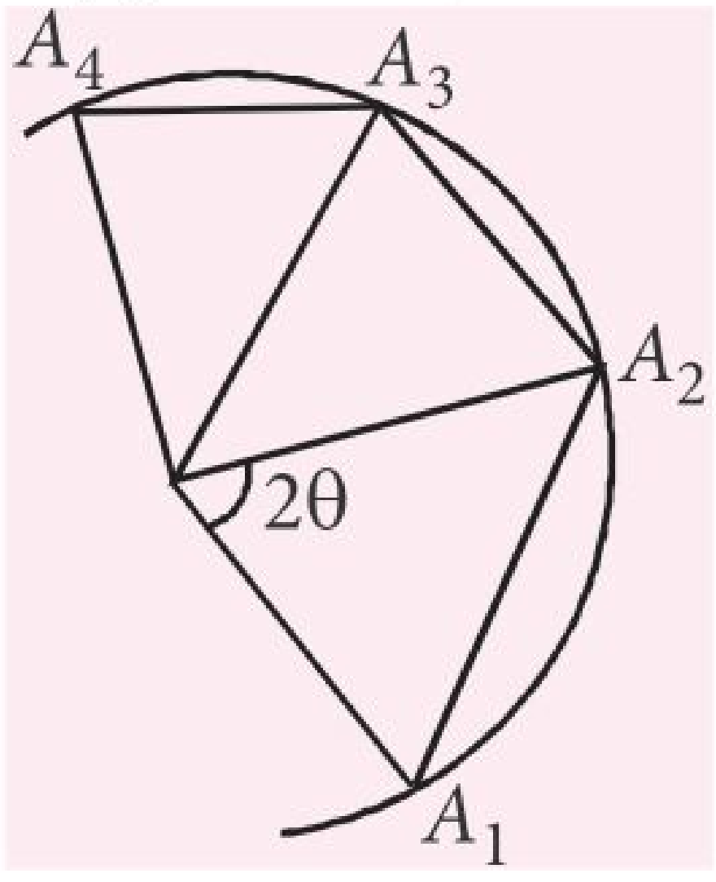
$$\Rightarrow \quad \frac{1}{\sin\theta} = \frac{\sin 3\theta + \sin 2\theta}{\sin 2\theta \sin 3\theta}$$

$$\Rightarrow \quad \sin 3\theta + \sin 2\theta = 2\sin 3\theta\cos\theta$$

$$= \sin 4\theta + \sin 2\theta$$

$$\Rightarrow \quad \sin 3\theta + \sin 4\theta$$

Since $3\theta, 4\theta \in [0,\pi]$, we get $3\theta + 4\theta = \pi = n\theta \Rightarrow n = 7$.



5. (a) Yes: simply rotate the plane 90° about some axis perpendicular to it. For example, in the xy-plane we could map each point (x, y) to the point $(y, -x)$.
(b) Suppose such a bijection existed. Label the three-dimensional space with x-axes, y-axes, and z-axes. Given any point $P = (x_0, y_0, z_0)$, we also view it as the vector p from (0,0,0) to (x_0, y_0, z_0). The given condition says that $(a - b) \cdot (f(a) - f(b)) = 0$ for any vectors a, b.
Assume without loss of generality that f maps the origin to itself otherwise, $g(p) = f(p) - f(0)$ is still a bijection and still satisfies the above equation. Plugging $b = (0,0,0)$ into the equation above, we have $a.f(a) = 0$ for all a. Then the above equation reduces to $a\cdot f(b) + b\cdot f(a) = 0$.
Given any vectors a, b, c and any reals m, n, we then have
$m(a\cdot f(b) + b\cdot f(a)) = 0$
$n(a\cdot f(c) + c\cdot f(a)) = 0$
$a\cdot f(mb + nc) + (mb + nc)\cdot f(a) = 0.$
Adding the first two equations and subtracting the third gives
$a\cdot(m f(b) + nf(c) - f(mb + nc)) = 0.$
Because this must be true for any vector a, we must have
$f(mb + nc) = m f(b) + nf(c).$
Therefore f is linear, and it is determined by how it transforms the unit vectors $i = (1, 0, 0)$, $j = (0, 1, 0)$, and $k = (0, 0, 1)$. If
$f(i) = (a_1, a_2, a_3)$, $f(j) = (b_1, b_2, b_3)$, and $f(k) = (c_1, c_2, c_3)$, then for a vector x we have

$$f(x) = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix} x.$$

Applying $f(a)\cdot a = 0$ with $a = i, j, k$, we have
$a_1 = b_2 = c_3 = 0.$
Then applying $a\cdot f(b) + b.\, f(a)$ with $(a, b) = (i, j)$, (j, k), (k, i) we have $b_1 = -a_2$, $c_2 = -b_3$, $c_1 = -a_3$.
Setting $k_1 = c_2$, $k_2 = -c_1$, and $k_3 = b_1$we find that
$f(k_1 i + k_2 j + k_3 k) = k_1 f(i) + k_2 f(j) + k_3 f(k) = 0$.
Because f is injective and $f(0) = 0$, this implies that $k_1 = k_2 = k_3 = 0$.
Then $f(x) = 0$ for all x, contradicting the assumption that f was surjective. Therefore our original assumption was false, and no such bijection exists.

6. Let the sides of the triangle have lengths $a \le b \le c$; let the angles opposite to them be A, B, C; let the semiperimeter be $s = \frac{1}{2}(a+b+c)$; and let the inradius be r. Without loss of generality, assume that the triangle have circumradius $R = \frac{1}{2}$ and that $a = \sin A$, $b = \sin B$, $c = \sin C$.
The area of the triangle equals both
$rs = \frac{1}{2} r\ (\sin A + \sin B + \sin C)$ and
$abc/uR = \frac{1}{2} \sin A \sin B \sin C$. Thus

$$r = \frac{\sin A \sin B \sin C}{\sin A + \sin B + \sin C} \text{ and } \frac{a}{r} = \frac{\sin A + \sin B + \sin C}{\sin B + \sin C}$$

Because $A = 180° - B - C$, $\sin A = \sin(B + C) = \sin B \cos C + \cos B \sin C$ and we also have

$$\frac{a}{r} = \cot B + \csc B + \cot C + \csc C$$

Note that $f(x)$ = cot$x$ + csc$x$ is a decreasing function along the interval $0^\circ < x < 90^\circ$. Now there are two cases; $B \le 60^\circ$, or $B > 60^\circ$.

If $B > 60^\circ$, because $C > B > 60^\circ$, the triangle with $A' = B' = C' = 60^\circ$ has a larger ratio a'/r'. Therefore we may assume that $B \le 60^\circ$. We may further assume that $A = B$ – otherwise, the triangle with angles

$$A' = B' = \frac{1}{2}(A + B) \le B \text{ and } C' = C$$

has a larger ratio a'/r' . Because $C < 90^\circ$ we have $45^\circ < A \le 60^\circ$. Now,

$$\frac{a}{r} = \frac{\sin A + \sin B + \sin C}{\sin B \sin C} = \frac{2\sin A + \sin(2A)}{\sin A \sin(2A)}$$

= 2csc(2A) + csc$A$.

Now csc$x$ has second derivative csc$x$(csc^2x + cot^2x), which is strictly positive when $0^\circ < x < 180^\circ$. Thus, both csc$x$ and csc(2x) are strictly convex along the interval $0^\circ < x < 90^\circ$. Therefore, $g(A)$ = 2csc(2A) + csc$A$, a convex function in A, is maximized in the interval $45^\circ \le A \le 60^\circ$ at one of the endpoints. Because $g(45^\circ) = 2 + \sqrt{2} < 2\sqrt{3} = g(60^\circ)$, it is maximized when $A = B = C = 60^\circ$.

Therefore the maximum ratio is $2\sqrt{3}$, attained with an equilateral triangle.

7. Let α, β, γ be the three roots. Without loss of generality, supose that $0 \le \alpha \le \beta \le \gamma$. We have $x - a = x + \alpha + \beta + \gamma \ge 0$ and $f(x) = (x - \alpha)(x - \beta)(x - \gamma)$. If $0 \le x \le \alpha$, then (applying the AM-GM inequality to obtain the first inequality below)

$$-f(x) = (\alpha - x)(\beta - x)(\gamma - x) \le \frac{1}{27}(\alpha + \beta + \gamma - 3x)^3$$

$$\le \frac{1}{27}(x + \alpha + \beta + \gamma)^3 = \frac{1}{27}(x - a)^3,$$

so that $f(x) \ge -\frac{1}{27}(x - a)^3$. Equality holds exactly when $\alpha - x = \beta - x = \gamma - x$ in the first inequality and $\alpha + \beta + \gamma - 3x = x + \alpha + \beta + \gamma$ in the second; that is, when $x = 0$ and $\alpha = \beta = \gamma$.

If $\beta \le x \le \gamma$, then (again applying the AM-GM inequality to obtain the first inequality below)

$$-f(x) = (x - \alpha)(x - \beta)(\gamma - x) \le \frac{1}{27}(x + \gamma - \alpha - \beta)^3$$

$$\le \frac{1}{27}(x + \alpha + \beta + \gamma)^3 = \frac{1}{27}(x - a)^3 \text{, so that again}$$

$f(x) \ge -\frac{1}{27}(x - a)^3$. Equality holds exactly when $x - \alpha = x - \beta = \gamma - x$ in the first inequaltiy and $x + \gamma - \alpha - \beta = x + \alpha + \beta + \gamma$; that is, when $\alpha = \beta = 0$ and $\gamma = 2x$.

Finally, when $\alpha < x < \beta$ or $x > \gamma$ then

$$f(x) > 0 \ge -\frac{1}{27}(x - a)^3$$

Thus, $\lambda = -\frac{1}{27}$ works. From the above reasoning we can find that λ must be at most $-\frac{1}{27}$ or else the inequality fails for the polynomial $f(x) = x^2(x - 1)$ at $x = \frac{1}{2}$. Equality occurs either when $\alpha = \beta = \gamma$ and $x = 0$; or when $\alpha = \beta = 0$, γ is any non-negative real, and $x = \frac{\gamma}{2}$.

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YOU ASK WE ANSWER

Do you have a question that you just can't get answered?
Use the vast expertise of our MTG team to get to the bottom of the question. From the serious to the silly, the controversial to the trivial, the team will tackle the questions, easy and tough. The best questions and their solutions will be printed in this column each month.

1. Given that for each $a \in (0, 1)$,

$$g(a) = \lim_{h\to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}\,dt \text{ exists. Find } g\left(\frac{1}{2}\right).$$

(Akshi Verma, Jharkhand)

Ans. We have, $g(a) = \lim_{h\to 0^+} \int_h^{1-h} t^{-a}(1-t)^{a-1}\,dt$

$$\therefore\quad g\left(\frac{1}{2}\right) = \lim_{h\to 0^+} \int_h^{1-h} t^{-1/2}(1-t)^{-1/2}\,dt$$

$$= \lim_{h\to 0^+} \int_h^{1-h} \frac{1}{\sqrt{t-t^2}}\,dt = \int_0^1 \frac{1}{\sqrt{\left(\frac{1}{2}\right)^2 - \left(t-\frac{1}{2}\right)^2}}\,dt$$

$$= \left[\sin^{-1}\left(\frac{t-\frac{1}{2}}{1/2}\right)\right]_0^1 = \sin^{-1}1 - \sin^{-1}(-1) = \pi$$

2. Show that

$$\int_0^\pi x^3 \ln(\sin x)\,dx = \frac{3\pi}{2}\int_0^\pi x^2 \ln(\sqrt{2}\sin x)\,dx.$$

(Neetu Singh, Delhi)

Ans. Let, $I = \int_0^\pi x^3 \ln(\sin x)dx$ ...(i)

$$\Rightarrow\quad I = \int_0^\pi (\pi - x)^3 \ln\{\sin(\pi - x)\}\,dx$$

$$\Rightarrow\quad I = \int_0^\pi (\pi^3 - 3\pi^2 x + 3\pi x^2 - x^3)\ln(\sin x)\,dx \quad \text{...(ii)}$$

Adding (i) and (ii), we get

$$2I = \pi^3\int_0^\pi \ln(\sin x)dx - 3\pi^2\int_0^\pi x\ln(\sin x)dx + 3\pi\int_0^\pi x^2\ln(\sin x)dx$$

$$= \pi^3(-\pi\ln 2) - 3\pi^2 I_1 + 3\pi\int_0^\pi x^2\ln(\sin x)dx \quad \text{...(iii)}$$

$$\left[\because \int_0^\pi \ln(\sin x)dx = -\pi\ln 2\right]$$

where, $I_1 = \int_0^\pi x\ln(\sin x)dx = \int_0^\pi (\pi - x)\ln(\sin x)dx$

Adding the integrals, we get

$$I_1 = \frac{\pi}{2}\int_0^\pi \ln(\sin x)\,dx = \frac{-\pi^2}{2}\ln 2$$

Hence, from (iii), we have

$$2I = -\pi^4\ln 2 - 3\pi^2\left(\frac{-\pi^2}{2}\ln 2\right) + 3\pi\int_0^\pi x^2\ln(\sin x)\,dx$$

$$i.e.,\quad I = \frac{\pi^4}{4}\ln 2 + \frac{3\pi}{2}\int_0^\pi x^2\ln(\sin x)dx$$

$$= \frac{3\pi}{2}\left[\frac{\pi^3}{3}\ln\sqrt{2} + \int_0^\pi x^2\ln(\sin x)dx\right]$$

$$= \frac{3\pi}{2}\left[\int_0^\pi x^2\ln\sqrt{2}dx + \int_0^\pi x^2\ln(\sin x)dx\right]$$

$$= \frac{3\pi}{2}\int_0^\pi x^2\ln(\sqrt{2}\sin x)\,dx$$

3. Solve the equation $\log_{(2x+3)}(6x^2 + 23x + 21) = 4 - \log_{(3x+7)}(4x^2 + 12x + 9)$

(Chirag Shetty, Maharashtra)

Ans. The given equation reduces to

$$\frac{\log(6x^2+23x+21)}{\log(2x+3)} + \frac{\log(4x^2+12x+9)}{\log(3x+7)} = 4$$

$$\Rightarrow\quad \frac{\log\left((2x+3)(3x+7)\right)}{\log(2x+3)} + 2\frac{\log(2x+3)}{\log(3x+7)} = 4$$

$$\Rightarrow\quad 1 + \frac{\log(3x+7)}{\log(2x+3)} + 2\frac{\log(2x+3)}{\log(3x+7)} = 4$$

$$\Rightarrow\quad y + \frac{2}{y} = 3 \qquad \left[\text{Putting } \frac{\log(3x+7)}{\log(2x+3)} = y\right]$$

$\Rightarrow$ $y^2 - 3y + 2 = 0$ gives $y = 1, 2$

$\therefore$ $3x + 7 = 2x + 3, (2x + 3)^2$

Solving $3x + 7 = 2x + 3$, we get $x = -4$

Solving $3x + 7 = (2x + 3)^2$, we get $4x^2 + 9x + 2 = 0$

$$\Rightarrow\quad x = \frac{-9\pm 7}{8} = -2, \frac{-1}{4}$$

Also, for $\dfrac{1}{\log(2x+3)}$ to be defined, $2x + 3 > 0$ and $2x + 3 \neq 1$

i.e., $x > \dfrac{-3}{2}$ and $x \neq -1$ and for $\dfrac{1}{\log(3x+7)}$ to be defined $3x + 7 > 0$ and $3x + 7 \neq 1$ *i.e.,* $x > \dfrac{-7}{3}$ and $x \neq -2$

Hence, the only value of x satisfying the above conditions is $\dfrac{-1}{4}$.

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CONCEPT MAP

PROBABILITY

Class XI

Basic Terms

- **Experiment :** An operation which results in some well defined outcomes.
- **Random Experiment :** An experiment whose outcome cannot be predicted with certainty.
- **Sample space :** Set of all possible outcomes.
- **Sample point :** Element of sample space.
- **Equally likely outcomes :** All outcomes with equal probability.

Algebra of Events

Let S be sample space and A, B be any event.

- **Complementary event :** $A' = S - A$
- **Event A or B :** Event $A \cup B$
- **Event A and B :** Event $A \cap B$
- **Event A and not B :** Event $A - B = A \cap B'$

Events

It is a subset of a sample space.

Types of Events

- **Impossible and Sure Events :** The empty set ϕ is called an impossible event and sample space S is called the sure event.
- **Simple Event :** Event having only one sample point.
- **Compound Event :** Event having more than one sample point.
- **Mutually Exclusive Events :** Event A and B are mutually exclusive, if $A \cap B = \phi$
- **Exhaustive and Mutually Exclusive Events :** Events $E_1, E_2, ..., E_n$ are mutually exclusive and exhaustive if $E_1 \cup E_2 \cup ... \cup E_n = S$ and $E_i \cap E_j = \phi\ \forall\ i \neq j$.

PROBABILITY

Probability

If the outcomes of an experiment are equally likely, probability of an event(E) is defined as

$$P(E) = \frac{\text{Number of outcomes favourable to } E}{\text{Total number of possible outcomes}}$$

In other words, if $S = \{x_1, x_2, ..., x_n\}$ and the number $P(x_i)$ associated with sample point x_i is such that

- $0 \le P(x_i) \le 1$
- $\sum P(x_i) = 1\ \forall\ x_i \in S$

  then $P(A) = \sum P(x_i)\ \forall\ x_i \in A$. The number $P(x_i)$ is called probability of the outcome x_i.

Addition Theorem

- If A and B are any two events of a sample space S, then the probability of occurrence of atleast one of the events A and B is given by

  $$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

  or in general for n events $A_1, A_2, ..., A_n$, we have

  $$P(A_1 \cup A_2 \cup ... \cup A_n) = \sum_{i=1}^{n} P(A_i) - \sum_{i<j} P(A_i \cap A_j) + \sum_{i<j<k} P(A_i \cap A_j \cap A_k) - ... + (-1)^{n-1} P(A_1 \cap A_2 \cap ... \cap A_n)$$
- If A and B are mutually exclusive events, then $P(A \cup B) = P(A) + P(B)$

Some Important Results

- P (exactly one of the events A, B occurs) $= P(A \cap \bar{B}) + P(\bar{A} \cap B) = P(A) + P(B) - 2P(A \cap B)$
- P (atleast one of the events A, B, C occurs) $= P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$
- P (atleast two of the events A, B, C occurs) $= P(A \cap B) + P(B \cap C) + P(C \cap A) - 2P(A \cap B \cap C)$
- P (exactly two of the events A, B, C occurs) $= P(A \cap B) + P(B \cap C) + P(C \cap A) - 3P(A \cap B \cap C)$
- P(exactly one of the events A, B, C occurs) $= P(A) + P(B) + P(C) - 2P(B \cap C) - 2P(C \cap A) - 2P(A \cap B) + 3P(A \cap B \cap C)$

DIFFERENTIABILITY

Class XII

CONCEPT MAP

Differentiability

A real valued function $f(x)$ is differentiable at $x = a$ if $\lim\limits_{x \to a} \frac{f(x) - f(a)}{x - a}$ exists finitely and it is denoted by $f'(a)$. In other words $f(x)$ is said to be differentiable at $x = a$ if its left hand derivative *i.e.*, $Lf'(a) = \lim\limits_{h \to 0} \frac{f(a-h) - f(a)}{-h}$ and right hand derivative *i.e.*, $Rf'(a) = \lim\limits_{h \to 0} \frac{f(a+h) - f(a)}{h}$ at $x = a$ are exist and equal.

Note : Every differentiable function is continuous but the converse is not necessarily true.

Derivative of Functions

- **Composite Function (Chain Rule) :** Let $y = f(t)$ and $t = g(x)$, then $\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$.
- **Implicit Function :** Here, we differentiate the function of type $f(x, y) = 0$, where x and y can't be expressed in terms of one another.
- **Parametric Function :** If $x = f(t)$ and $y = g(t)$, then $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{g'(t)}{f'(t)}, f'(t) \neq 0.$
- **Logarithmic Differentiation :** If $y = u^v$, where u, v are functions of x, then

  $$\log y = v \log u \Rightarrow \frac{d}{dx}(u^v) = u^v \left[\frac{v}{u}\frac{du}{dx} + \log u \frac{dv}{dx}\right].$$

Second Order Derivative

Let $y = f(x)$, then $\frac{dy}{dx} = f'(x)$. If $f'(x)$ is differentiable, then $\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(f'(x))$ or $\frac{d^2y}{dx^2} = f''(x)$ is the second order derivative.

Leibnitz Formula

The n^{th} derivative of the product of two functions : If u, v be two functions possessing derivative of the n^{th} order, then

$$(uv)_n = u_n v + {}^nC_1\, u_{n-1} v_1 + {}^nC_2 u_{n-2} v_2 + ... + {}^nC_r\, u_{n-r} v_r + ... + {}^nC_n u v_n$$

$$= \sum_{r=0}^{n} C(n, r) u_{n-r} v_r, \text{ where } {}^nC_r = \frac{n!}{(n-r)!r!}.$$

Algebra of Derivatives

- $(u \pm v)' = u' \pm v'$
- $(uv)' = uv' + u'v$
- $\left(\frac{u}{v}\right)' = \frac{u'v - uv'}{v^2}, v \neq 0$
- $(\alpha u)' = \alpha u'$, where α is any scalar

Some Standard Derivatives

- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\cos^{-1} x) = \frac{-1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- $\frac{d}{dx}(\cot^{-1} x) = \frac{-1}{1+x^2}$
- $\frac{d}{dx}(\sec^{-1} x) = \frac{1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}(\mathrm{cosec}^{-1} x) = \frac{-1}{|x|\sqrt{x^2-1}}$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx} a^x = a^x \log a, a > 0$ and $a \neq 1$
- $\frac{d}{dx}(\log x) = \frac{1}{x}, x > 0$
- $\frac{d}{dx}(\ln(x + \sqrt{x^2+1})) = \frac{1}{\sqrt{x^2+1}}$
- $\frac{d}{dx}(\ln(x + \sqrt{x^2-1})) = \frac{1}{\sqrt{x^2-1}}$
- $\frac{d^n}{dx^n} \cos(ax+b) = a^n \cos\left(ax + b + \frac{n\pi}{2}\right)$
- $\frac{d^n}{dx^n} \sin(ax+b) = a^n \sin\left(ax + b + \frac{n\pi}{2}\right)$
- (i) $\frac{d^n}{dx^n} e^{ax} \cos(bx+c) = r^n e^{ax} \cos(bx + c + n\theta)$

  (ii) $\frac{d^n}{dx^n} e^{ax} \sin(bx+c) = r^n e^{ax} \sin(bx + c + n\theta)$,

  where $a = r\cos\theta, b = r\sin\theta, r = \sqrt{a^2+b^2}, \theta = \tan^{-1}\frac{b}{a}$

Some Important Theorems

- **Rolle's Theorem :** If $f(x)$ is a real valued function defined on $[a, b]$ is continuous on $[a, b]$ and differentiable on (a, b) such that $f(a) = f(b)$, then $\exists$ atleast one $c \in (a, b)$ such that $f'(c) = 0$.
- **Lagrange's Mean Value Theorem :** If $f(x)$ is a real valued function defined on $[a, b]$ is continuous on $[a, b]$ and differentiable on (a, b), then $\exists$ atleast one $c \in (a, b)$ such that $f'(c) = \frac{f(b) - f(a)}{b - a}$.

JEE WORK OUTS

One or More Than One Option(s) Correct Type

1. The relation R defined on a set $A = \{1, 2, 3, 4, 5\}$ by $R = \{(a, b) : |a^2 - b^2| < 16\}$ is given by

(a) $R = \{(1, 1), (2, 1), (3, 1), (4, 1), (2, 3)\}$
(b) $R = \{(2, 2), (3, 2), (4, 2), (2, 4)\}$
(c) $R = \{(3, 3), (4, 3), (5, 4), (3, 4)\}$
(d) none of these

2. If $f : R \to R$ and $g : R \to R$ are one to one, real valued functions then $\phi(x) = [f(x) + f(-x)][g(x) - g(-x)]$ is

(a) an even function
(b) an odd function
(c) $\phi'(x)$ is a odd function
(d) $\phi'(x)$ is an even function

3. If 3 arithmetic means, 3 geometric means and 3 harmonic means are inserted between 1 and 5, then the cubic equation whose roots are first A.M., second G.M. and third H.M. between 1 and 5, is

(a) $x^3 - \left(\frac{9}{2} + \sqrt{5}\right)x^2 + \left(\frac{9\sqrt{5}}{2} + 5\right)x - 5\sqrt{5} = 0$

(b) $x^3 + \left(\frac{9}{2} + \sqrt{5}\right)x^2 - \left(\frac{9\sqrt{5}}{2} + 5\right)x - 5\sqrt{5} = 0$

(c) $x^3 + \left(\frac{9}{2} - \sqrt{5}\right)x^2 - \left(\frac{9\sqrt{5}}{2} - 5\right)x + 5\sqrt{5} = 0$

(d) none of these

4. $\lim\limits_{x\to 0} \frac{\log(a+x) - \log a}{x} + k \lim\limits_{x\to e} \frac{\log x - 1}{x - e} = 1$, then

(a) $k = e\left(1 - \frac{1}{a}\right)$
(b) $k = e(1 + a)$
(c) $k = e(2 - a)$
(d) The equality is not possible

5. $\lim\limits_{x\to a} f(x)\cdot g(x)$ exists, if

(a) $\lim\limits_{x\to a} f(x)$ and $\lim\limits_{x\to a} g(x)$ exist
(b) $\lim\limits_{x\to a} f(x)^{g(x)}$ exist
(c) $\lim\limits_{x\to a} \frac{f(x)}{g(x)}$ exist
(d) $\lim\limits_{x\to a} f(x) g\left(\frac{1}{x}\right)$ exist

6. If $f(x) = 2x^3 - 15x^2 + 24x$ and

$$g(a) = \int_0^a f(x)dx + \int_0^{5-a} f(x)dx \quad (1 < a < 4), \text{ then}$$

(a) g is increasing in $\left(1, \frac{5}{2}\right)$
(b) g is decreasing in $\left(\frac{5}{2}, 4\right)$
(c) g is increasing in $(0, 5)$
(d) None of these

7. If $f(x)$, $g(x)$ (where $x > 1$) are non–negative and non–positive functions respectively, such that $f'(x) \le \alpha f(x)$, $g'(x) \ge \beta g(x)$ for some $\alpha, \beta > 0$ and $f(1) = 0$, $g(1) = 0$, then

(a) $\frac{f(e) + f(\pi)}{f^2(e) + f^2(\pi) - 4} = 0$

(b) $\frac{f(e) + f(\pi)}{f^2(e) + f^2(\pi) - 4} = -4$

(c) $\frac{g(\sqrt{e}) + g(\sqrt{\pi})}{g(\sqrt[3]{e}) + g(\sqrt[3]{\pi}) - 3} = 0$

(d) $\frac{g(\sqrt{e}) + g(\sqrt{\pi})}{g(\sqrt[3]{e}) + g(\sqrt[3]{\pi}) - 3} = -3$

8. Let $f(x)=\int_0^x (u-1)(u-2)^2\,du$, then for the function $f(x)$

(a) $(1, -3)$ is a point of minimum

(b) $(2, -4/3)$ is a point of inflection

(c) $\left(1, -\frac{17}{12}\right)$ is a point of minimum

(d) $\left(\frac{4}{3}, -\frac{112}{81}\right)$ is a point of inflection.

9. POQ is a straight line through the origin O, P and Q represent the complex numbers $a + ib$ and $c + id$ respectively and $OP = OQ$, then

(a) $|a + ib| = |c + id|$

(b) $a + c + b + d = 0$

(c) $\arg(a + ib) = \arg(c + id)$

(d) None of these

10. Let a be a complex number such that $|a| < 1$ and $z_1, z_2, ...$ be vertices of a polygon such that $z_k = 1 + a + a^2 + ... + a^{k-1}$. Then the vertices of the polygon lie within a circle

(a) $|z - a| = a$ (b) $\left|z-\frac{1}{1-a}\right| = |1-a|$

(c) $\left|z-\frac{1}{1-a}\right|=\frac{1}{|1-a|}$ (d) $|z-(1-a)| = |1-a|$

11. If $\frac{|z-2|}{|z-3|}=2$ represents a circle, then its radius is equal to

(a) 1 (b) 1/3 (c) 3/4 (d) 2/3

12. The points $1 + 3i$, $5 + i$ and $3 + 2i$ in the complex plane are

(a) vertices of a right angled triangle

(b) collinear

(c) vertices of an obtuse angled triangle

(d) vertices of an equilateral triangle

13. The values of 'a' for which the roots of the equation $(a + 1)x^2 - 3ax + 4a = 0\ (a \neq -1)$ are real and greater than 1 is

(a) $\left[-\frac{10}{7}, 1\right]$ (b) $\left[-\frac{12}{7}, 0\right]$

(c) $\left[-\frac{16}{7}, -1\right)$ (d) $\left[-\frac{16}{7}, 0\right)$

14. If $f(x)$ is a polynomial of least degree such that $f(r)=\frac{1}{r},\ r=1, 2, 3, ..., 9$, then $f(10) =$

(a) 1 (b) 1/2 (c) 1/10 (d) 1/5

15. If the equations $x^2 + 2\lambda x + \lambda^2 + 1 = 0$, $\lambda \in R$ and $ax^2 + bx + c = 0$, where a, b, c are lengths of sides of triangle have a common root, then the possible range of values of λ is

(a) $(0, 2)$ (b) $(\sqrt{3}, 3)$

(c) $(2\sqrt{2}, 3\sqrt{2})$ (d) $(0, \infty)$

16. If $a_k=\frac{1}{k(k+1)}$, for $k=1,2,3,....,n$, then $\left(\sum_{k=1}^{n} a_k\right)^2 =$

(a) $\frac{n}{n+1}$ (b) $\frac{n^2}{(n+1)^2}$

(c) $\frac{n^4}{(n+1)^4}$ (d) $\frac{n^6}{(n+1)^6}$

17. The ratio of sum of first three terms of a G.P. to the sum of first six terms is 64 : 91, the common ratio of G.P. is

(a) 1/4 (b) 3/4 (c) 5/4 (d) 7/4

18. If the roots of equation $x^3 + bx^2 + cx - 1 = 0$ form an increasing G.P., then

(a) one of the root is 1

(b) $b \in (-\infty, -3)$

(c) $b + c = 0$

(d) one root is <1 and other root is > 1

19. If a, b, c are positive rational numbers satisfies $a > b > c$ and the quadratic equation $(a + b - 2c)x^2 + (b + c - 2a)x + (c + a - 2b) = 0$ has a root in the interval $(-1, 0)$, then

(a) both roots of the equation are rational

(b) $c + a < 2b$

(c) the equation $ax^2 + 2bx + c = 0$ has both negative real roots.

(d) the equation $cx^2 + 2ax + b = 0$ has both negative real roots.

20. If $(x + 1)^4 + (x + 3)^4 = 16$, then value/values of x are

(a) -1 (b) -3

(c) $-2 - i\sqrt{7}$ (d) $-2 + i\sqrt{7}$

Comprehension Type

Paragraph for Q. No. 21 to 23

Let $f(x) = ax^2 + bx + c$; $a, b, c \in R$.

It is given that $|f(x)| \leq 1,\ \forall\ |x| \leq 1$

21. The possible value of $|a + b|$ if $4a^2 + 3b^2$ is maximum is

(a) 1 (b) 0 (c) 2 (d) 3

22. The possible values of $|a + b|$ if $\frac{8}{3}a^2 + 2b^2$ is maximum is given by

(a) 1 (b) 0 (c) 2 (d) 3

23. The possible maximum value of $\frac{8}{3}a^2+2b^2$ is given by

(a) 32 (b) $\frac{32}{3}$ (c) $\frac{2}{3}$ (d) $\frac{16}{3}$

Paragraph for Q. No. 24 to 26

In the evaluation of limits in the following passage, one may use one or the other of the following results:

- If $f(x)$ and $g(x)$ are functions defined in some deleted neighbourhood N of 'a' such that $g(x)$ never vanishes in N and $\lim\limits_{x\to a}\frac{f(x)}{g(x)}$ exists, then $\lim\limits_{x\to a}g(x)=0$ implies $\lim\limits_{x\to a}f(x)=0$.
- If $f(x)\to 1$ as $x\to a$, $g(x)\to+\infty$ as $x\to a$ and $\lim\limits_{x\to a}\{f(x)\}^{g(x)}$ exists, then this limit equals $e^{\lim\limits_{x\to a}g(x)\{f(x)-1\}}$.

24. If $\lim\limits_{x\to 0}\frac{\sin ax-\log_e(e^x\cos x)}{x\sin bx}=\frac{1}{2}$, then

(a) $a=1, b=1$ (b) $a=-1, b=-1$

(c) $a=-1, b=1$ (d) $a=-1, b=1/2$

25. Let $f(x)$ be a function that is defined in a deleted neighbourhood of '0' such that $\lim\limits_{x\to 0}\frac{f(x)}{x}=1$. If $\lim\limits_{x\to 0}\frac{x(1+a\cos x)-b\sin x}{\{f(x)\}^3}=1$, then

(a) $a=-\frac{3}{2},b=-\frac{1}{2}$ (b) $a=\frac{1}{2},b=\frac{3}{2}$

(c) $a=-\frac{5}{2},b=-\frac{3}{2}$ (d) $a=\frac{3}{2},b=\frac{5}{2}$

26. Let $L=\lim\limits_{x\to 0}\frac{a-\sqrt{a^2-x^2}-\frac{x^2}{2}}{x^2\sin^2 x}, a>0$. If L is finite, then L =

(a) 1/4 (b) –1/4 (c) 1/8 (d) –1/8

Paragraph for Q. No. 27 to 28

Let $f(x)={}^nC_x p^x q^{n-x}; x=0,1,2,.....,n$, where $p+q=1$, $0<p<1$.

Here ${}^nC_x=\frac{n!}{x!(n-x)!}$ and $n! = 1\times 2\times 3\times \ldots \times n$,

$np=\lambda$ (finite) when $p\to 0$ and $n\to\infty$.

27. $\lim\limits_{n\to\infty}\sum\limits_{x=0}^{n}x\,f(x)=$

(a) λ (b) $\lambda/2$ (c) 0 (d) 1

28. $\lim\limits_{n\to\infty}\sum\limits_{x=0}^{n}x^2 f(x)=$

(a) $1+\lambda$ (b) $\lambda+\lambda^2$ (c) λ^2 (d) 0

Matrix Match Type

29. z_1, z_2, z_3 are vertices of a triangle. Match the condition in column-I with type of triangle in column-II.

Column-I		Column-II	
A.	$z_1^2+z_2^2+z_3^2=z_2z_3+z_3z_1+z_1z_2$	P.	right angled triangle
B.	$\text{Re}\left(\frac{z_3-z_1}{z_3-z_2}\right)=0$	Q.	obtuse angled triangle
C.	$\text{Re}\left(\frac{z_3-z_1}{z_3-z_2}\right)<0$	R.	isosceles and right angled triangle
D.	$\frac{z_3-z_1}{z_3-z_2}=i$	S.	equilateral triangle

30. Match the following :

Column-I		Column-II	
A.	If $x+\frac{1}{x}=\sqrt{3}$, then $x^{2018}+\frac{1}{x^{2018}}$ equals to	P.	2
B.	Let $z=1+i\sqrt{3}$, then $\frac{z^6+(\bar{z})^6}{2^5}$ equals to	Q.	1
C.	If 1, α_1, α_2, …, α_{n-1} are the n n^{th} roots of unity. If n is an odd natural number, then the value of $(1+\alpha_1)(1+\alpha_2)(1+\alpha_3)\ldots(1+\alpha_{n-1})$ equals to	R.	3
D.	If $z-\frac{1}{z}=i$, then $z^{2018}+\frac{1}{z^{2018}}$ equals to	S.	4
E.	If $\lvert z+4i\rvert\le 2$, then minimum value of $\lvert z+3\rvert$ =		
F.	If $x=-2-i\sqrt{3}$, then value of $2x^4+5x^3+7x^2-x+37$ =		

31. Match the following:

Column- I		Column- II	
A.	If f is a function such that $f(0)=2$, $f(1)=3$ and $f(x+2)=2f(x)-f(x+1)$, then $f(5)$ is equal to	P.	4
B.	If $f(x)=\begin{cases}x^2, & \text{for } x\ge 0\\ x, & \text{for } x<0\end{cases}$, then $f(\sqrt{13})=$	Q.	3

C.	If $f(x) + 2f(1 - x) = x^2 + 2$ for all $x \in R$, then $f(5)$ is	R.	12
D.	If $f(x)=\dfrac{4^x}{4^x+2}$ for all $x \in R$, then $\sum\limits_{k=1}^{6} f\left(\dfrac{k}{7}\right)$ equals	S.	2
		T.	13

Numerical Answer Type

32. The value of sec40° + sec80° + sec160° will be______.

33. If 1 lies between the roots of the equation $y^2 - my + 1 = 0$ and $[x]$ denotes the greatest integer less than or equal to x, then the value of $\left[\left(\dfrac{4|x|}{|x|^2+16}\right)^m\right]+3$ is equal to ______.

34. If a curve is represented parametrically by the equations : $x = 4t^3 + 3$ and $y = 4 + 3t^4$ and $\dfrac{d^2x}{dy^2}\Big/\left(\dfrac{dx}{dy}\right)^n$ is constant, then the value of n is equal to ______.

35. An infinite number of tangents can be drawn from (1, 2) to the circle $x^2 + y^2 - 2x - 4y + \lambda = 0$, then λ = ______.

36. The number of triplet (x, y, z), where x, y, z are positive integer satisfying the equation

$$\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix} = 30 \text{ is \_\_\_\_\_\_.}$$

37. If $\vec{a}, \vec{b}, \vec{c}$ be any three unit vectors such that $3\vec{a} + 4\vec{b} + 5\vec{c} = 0$, then $\vec{a} \cdot \vec{b}$ equals ______.

38. If α, β, γ are the roots of the equation $x^3 + px + q = 0$, then the value of the determinant $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix}$ is ______.

39. If $\sum\limits_{r=0}^{n}\left\{\dfrac{{}^nC_{r-1}}{{}^nC_r + {}^nC_{r-1}}\right\}^3 = \dfrac{25}{24}$, then n is equal to ______.

40. For each positive integer n, let

$$s_n = \frac{3}{1\cdot2\cdot4} + \frac{4}{2\cdot3\cdot5} + \frac{5}{3\cdot4\cdot6} + \ldots.. + \frac{n+2}{n(n+1)(n+3)}$$

Then $\lim\limits_{n\to\infty} s_n = \dfrac{a}{b}$ (a, b are coprimes), where $a + b$ = ______.

SOLUTIONS

1. (d) : We have, $(a, b) \in R \Leftrightarrow |a^2 - b^2| < 16$

When $a = 1 \Rightarrow |1 - b^2| < 16 \Rightarrow |b^2 - 1| < 16$

$\Rightarrow -15 < b^2 < 17 \Rightarrow 0 < b^2 < 17 \Rightarrow b = 1, 2, 3, 4$

When $a = 2 \Rightarrow |4 - b^2| < 16 \Rightarrow |b^2 - 4| < 16$

$\Rightarrow -12 < b^2 < 20 \Rightarrow 0 < b^2 < 20 \Rightarrow b = 1, 2, 3, 4$

When $a = 3 \Rightarrow |9 - b^2| < 16 \Rightarrow |b^2 - 9| < 16$

$\Rightarrow -7 < b^2 < 25 \Rightarrow 0 < b^2 < 25 \Rightarrow b = 1, 2, 3, 4$

When $a = 4 \Rightarrow |16 - b^2| < 16 \Rightarrow |b^2 - 16| < 16$

$\Rightarrow 0 < b^2 < 32 \Rightarrow b = 1, 2, 3, 4, 5$

When $a = 5 \Rightarrow |25 - b^2| < 16 \Rightarrow |b^2 - 25| < 16$

$\Rightarrow 9 < b^2 < 41 \Rightarrow b = 4, 5.$

$$\therefore\ R=\left\{\begin{matrix}(1,1),(1,2),(1,3),(1,4),(2,1),(2,2),(2,3),(2,4),\\(3,1),(3,2),(3,3),(3,4),(4,1),(4,2),(4,3),(4,4),\\(4,5),(5,4),(5,5)\end{matrix}\right\}$$

2. (b, d) : Given $\phi(x) = [f(x) + f(-x)]\,[g(x) - g(-x)]$

$\therefore\ \phi(-x) = [f(-x) + f(x)][g(-x) - g(x)] = -\phi(x)$

$\therefore\ \phi(x)$ is an odd function.

3. (a) : Let a = First A.M. between 1 and 5

$\Rightarrow\ a = 1 + \dfrac{5-1}{3+1} = 2$

b = Second G.M. between 1 and 5

$\Rightarrow\ b = 1\left(\dfrac{5}{1}\right)^{2/4} = \sqrt{5}$

c = Third H.M. between 1 and 5

$\Rightarrow\ c = \dfrac{1}{1+3\times\left(\dfrac{1}{5}-1\right)\times\dfrac{1}{4}} = \dfrac{5}{2}$

$\therefore\ a+b+c = 2+\sqrt{5}+\dfrac{5}{2},\ ab+bc+ca = 2\sqrt{5}+\dfrac{5\sqrt{5}}{2}+5$

and $abc = 5\sqrt{5}$

The cubic equation having a, b, c as its roots is

$x^3 - (a + b + c)x^2 + (ab + bc + ca)x - abc = 0$

4. (a) : Let $f(x) = \log x \Rightarrow f'(x) = \dfrac{1}{x}$

Therefore, given function $= f'(a) + kf'(e) = 1$

$\Rightarrow\ \dfrac{1}{a} + \dfrac{k}{e} = 1 \ \Rightarrow\ k = e\left(\dfrac{a-1}{a}\right)$

5. (a) : The limit of product exists if the product of limits exist.

6. (a, b) : Here $f(x) = 2x^3 - 15x^2 + 24x$

$\therefore\ f'(x) = 6x^2 - 30x + 24 = 6(x - 4)(x - 1)$

$\Rightarrow\ f'(x) < 0$ for all $x \in (1, 4)$

Also, $g(a) = \int_0^a f(x)dx + \int_0^{5-a} f(x)dx$

$\Rightarrow\ g'(a) = f(a) - f(5 - a)$

If $a < 5 - a \ \Rightarrow\ a < \frac{5}{2}$

$g'(a) > 0 \Rightarrow g$ is increasing in $\left(1, \frac{5}{2}\right)$

And if $a > \frac{5}{2}$, $g'(a) < 0$

$\Rightarrow\ g$ is decreasing in $\left(\frac{5}{2}, 4\right)$.

7. (a, c) : Here, $f(x) \geq 0$

and $f'(x) - \alpha f(x) \leq 0$ (Given)

$\Rightarrow\ \frac{d}{dx}[e^{-\alpha x} f(x)] \leq 0\ \forall x > 1$

$\Rightarrow\ f(x) \leq 0\ \forall\ x > 1$

$\because\ f(1) = 0$

$\Rightarrow\ f(x) = 0\ \forall\ x > 1$

Similarly, $g(x) = 0\ \forall\ x > 1$

Now $\frac{f(e) + f(\pi)}{f^2(e) + f^2(\pi) - 4} = 0$ and $\frac{g(\sqrt{e}) + g(\sqrt{\pi})}{g(\sqrt[3]{e}) + g(\sqrt[3]{\pi}) - 3} = 0$

8. (b, c, d) : Given $f(x) = \int_0^x (u - 1)(u - 2)^2 du$

$\Rightarrow\ f'(x) = (x - 1)(x - 2)^2$

Put $f'(x) = 0\ \Rightarrow\ x = 1, 2$

At $x = 1$, $f'(x)$ changes its sign from –ve to +ve, so $f(x)$ is minimum at $x = 1$ and $f(1) = -\frac{17}{12}$.

Now, $f''(x)$ changes its sign at $x = 2, \frac{4}{3}$.

So, $x = 2, x = \frac{4}{3}$ are the point of inflexion.

$f(2) = -\frac{4}{3}, f\left(\frac{4}{3}\right) = -\frac{112}{81}$.

9. (a) : It is given that $OP = OQ$

$\therefore\ |OP| = |OQ|\ \Rightarrow\ |a + ib| = |c + id|$

10. (c) : We have, $z_k = 1 + a + a^2 + \ldots\ldots + a^{k-1} = \frac{1 - a^k}{1 - a}$

$\Rightarrow\ z_k - \frac{1}{1 - a} = \frac{-a^k}{1 - a}$

$\Rightarrow\ \left|z_k - \frac{1}{1 - a}\right| = \frac{|a^k|}{|1 - a|} = \frac{|a|^k}{|1 - a|} < \frac{1}{|1 - a|}$

$\Rightarrow\ z_k$ lies within $\left|z - \frac{1}{1 - a}\right| = \frac{1}{|1 - a|}$

11. (d) : Given, $\frac{|z - 2|}{|z - 3|} = 2$

$\Rightarrow\ \sqrt{(x - 2)^2 + y^2} = 2\sqrt{(x - 3)^2 + y^2}$

$\Rightarrow\ 3x^2 + 3y^2 - 20x + 32 = 0$

$\Rightarrow\ x^2 + y^2 - \frac{20}{3}x + \frac{32}{3} = 0$ ...(i)

We know that, standard equation of circle is

$x^2 + y^2 + 2gx + 2fy + c = 0$ ...(ii)

Comparing (i) and (ii), we get

$2g = -\frac{20}{3}\ \Rightarrow\ g = -\frac{10}{3}, f = 0, c = \frac{32}{3}$

Hence, Radius $= \sqrt{g^2 + f^2 - c} = \sqrt{\frac{100}{9} - \frac{32}{3}} = \sqrt{\frac{4}{9}} = \frac{2}{3}$

12. (b) : Let $z_1 = 1 + 3i$, $z_2 = 5 + i$ and $z_3 = 3 + 2i$

Then area of triangle formed by z_1, z_2 and z_3

$$= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 5 & 1 & 1 \\ 3 & 2 & 1 \end{vmatrix} = 0$$

Hence, z_1, z_2 and z_3 are collinear.

13. (c) : $D = 9a^2 - 16a(a + 1) \geq 0$,

$\Rightarrow\ a(7a + 16) \leq 0$ ...(i)

Also, $x_1 > 1$, $x_2 > 1$, where $x_1 + x_2 = \frac{3a}{a + 1}, x_1x_2 = \frac{4a}{a + 1}$

$\Rightarrow\ x_1 + x_2 - 2 > 0$ and $(x_1 - 1)(x_2 - 1) > 0$

$\Rightarrow\ \frac{a - 2}{a + 1} > 0$ ...(ii)

and $\frac{2a + 1}{a + 1} > 0$ ...(iii)

Solving (i), (ii) and (iii), we get

$-\frac{16}{7} \leq a < -1$

14. (d) : Here, $xf(x) - 1 = 0$ has roots 1, 2, 3,....., 9

$\therefore\ xf(x) - 1 = A(x - 1)(x - 2)\ \ldots.(x - 9)$ ...(i)

Put $x = 0$ in (i) $\Rightarrow\ A = \frac{1}{9!}$

Put $x = 10$ in (i) $\Rightarrow 10f(10) - 1 = 1$

$\Rightarrow f(10) = \frac{1}{5}$

15. (a) : $(x + \lambda)^2 + 1 = 0$ has clearly imaginary roots. So, both roots of the equations are common.

$\therefore \frac{a}{1} = \frac{b}{2\lambda} = \frac{c}{\lambda^2 + 1} = k$ (say)

Then $a = k$, $b = 2\lambda k$, $c = (\lambda^2 + 1)k$

$\because$ a, b, c are lengths of sides of triangle [Given]

$\therefore a + b > c \Rightarrow 2\lambda + 1 > \lambda^2 + 1 \Rightarrow \lambda^2 - 2\lambda < 0 \Rightarrow \lambda \in (0, 2)$

The other conditions will also imply same relation.

16. (b) : $\left(\sum_{k=1}^{n} a_k\right)^2 = \left[\sum_{k=1}^{n}\left(\frac{1}{k} - \frac{1}{k+1}\right)\right]^2$

$= \left(\frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \ldots\ldots + \frac{1}{n} - \frac{1}{n+1}\right)^2 = \frac{n^2}{(n+1)^2}$

17. (b) : Given, $\frac{S_3}{S_6} = \frac{64}{91} = \frac{a(r^3 - 1)}{a(r^6 - 1)}$

$\Rightarrow \frac{(r^3 - 1)}{(r^3 + 1)(r^3 - 1)} = \frac{64}{91} \Rightarrow r^3 = \frac{27}{64} \therefore r = \frac{3}{4}$

18. (a, b, c, d) : Suppose the roots are $\frac{\alpha}{r}$, α, αr, where $\alpha > 0$ and $r > 1$.

$\therefore \frac{\alpha}{r} + \alpha + \alpha r = -b$ (sum of roots) ...(i)

Sum of the product of roots taken two at a time

$= \alpha^2\left(\frac{1}{r} + r + 1\right) = c$ ...(ii)

Product of roots taken all at a time $= \frac{\alpha}{r}(\alpha)(\alpha r) = 1$ ...(iii)

$\Rightarrow \alpha^3 = 1 \Rightarrow \alpha = 1$ (Other values are imaginary)

$\therefore$ Sum of roots $= \frac{1}{r} + 1 + r = -b$

$\Rightarrow \left(\frac{1}{\sqrt{r}} - \sqrt{r}\right)^2 + 3 = -b$

$\Rightarrow \left(\frac{1}{\sqrt{r}} - \sqrt{r}\right)^2 = -(b + 3) > 0$

$\Rightarrow b < -3 \Rightarrow b \in (-\infty, -3)$

Again from sum of the product of roots taken two at a time and using $\alpha = 1$, we have $\frac{1}{r} + r + 1 = c$ and $\frac{1}{r} + r + 1 = -b$

$\Rightarrow b = -c \Rightarrow b + c = 0$

Now, $r > 1$ and roots are $\frac{\alpha}{r}$, α, αr

$\Rightarrow \frac{\alpha}{r} = \frac{1}{r} < 1$ and $\alpha r = r > 1$

$\therefore$ One root is < 1 and other root is > 1.

19. (a, b, c, d) : Since, we know that if sum of the coefficients of a quadratic equation is zero, then roots of the equation be rational and one of the root be 1 and other root be $\frac{\text{constant term}}{\text{coefficient of } x^2}$.

Here, sum of the coefficients is $(a + b - 2c) + (b + c - 2a) + (c + a - 2b) = 0$.

$\therefore$ One of the root is 1 and other root is $\frac{c + a - 2b}{a + b - 2c}$, which is rational as a, b, c are rational. Thus both roots of the equation are rational.

Also, $a > b > c \Rightarrow a - b > 0, a - c > 0, b - c > 0$ ...(i)

Again, root of the equation lies in the interval $(-1, 0)$

$\therefore f(-1) f(0) < 0$, where $f(0) = c + a - 2b$ and $f(-1) = 2(2a - b - c)$

$\Rightarrow 2(2a - b - c)(c + a - 2b) < 0$

$\Rightarrow [(a - c) + (a - b)](c + a - 2b) < 0$

$\Rightarrow (c + a - 2b) < 0$ (Using (i))

$\Rightarrow c + a < 2b$ ...(ii)

Now, in order to check that equation $ax^2 + 2bx + c = 0$ has both roots real and negative we need to show $D > 0$.

Sum of roots is < 0, sum is clearly < 0 as $S = -\frac{2b}{a} < 0$ as $a, b > 0$.

Again, $(c + a) < 2b$ [From (ii)]

$\Rightarrow 2b > (c + a)$

$\Rightarrow 4b^2 > (c + a)^2$

$\Rightarrow 4b^2 > c^2 + a^2 + 2ac$

$\Rightarrow 4b^2 - 4ac > c^2 + a^2 - 2ac$

$\Rightarrow 4b^2 - 4ac > (c - a)^2 > 0$

$\Rightarrow D > 0$ and $a, b, c > 0$ and $a > b > c$, $S = \frac{-2b}{a} < 0$

Product of roots $= c/a > 0$

$\therefore$ Roots of the equation $ax^2 + 2bx + c = 0$ are real and negative.

Similarly, discriminant of $cx^2 + 2ax + b = 0$ is $D = 4a^2 - 4bc = 4(a^2 - bc) > 0$ as $a > b > c$, then $a^2 > bc$

$\therefore D > 0$ and sum of roots $= \frac{-2a}{c} < 0$

and product of roots $= \frac{b}{c} > 0$

Thus, roots of $cx^2 + 2ax + b = 0$ are real and negative.

20. (a, b, c, d) : The given equation is of the form $(x \pm a)^4 + (x \pm b)^4 = c$

Substitute $y = x \pm \dfrac{a+b}{2}$ for simple form

Given equation is $(x + 1)^4 + (x + 3)^4 = 16$ ...(i)

Putting $y = x + \dfrac{1+3}{2}$ *i.e.*, $y = x + 2$

$\therefore\ x + 1 = y - 1$ and $x + 3 = y + 1$

$\therefore$ From (i) we have

$(y - 1)^4 + (y + 1)^4 = 16 \Rightarrow 2(y^4 + 6y^2 + 1) = 16$

$\Rightarrow y^4 + 6y^2 - 7 = 0 \Rightarrow (y^2 + 7)(y^2 - 1) = 0$

$\Rightarrow y = \pm 1, y = \pm i\sqrt{7}$

$\Rightarrow x + 2 = \pm 1, x + 2 = \pm i\sqrt{7}$

$\Rightarrow x = \pm 1 - 2, x = -2 \pm i\sqrt{7}$

$\Rightarrow x = -1, -3, x = -2 \pm i\sqrt{7}$

(21-23) : $|f(1) - f(0)| \le 2$

$\Rightarrow |a + b| \le 2 \Rightarrow (a + b)^2 \le 4$

Also, $|f(-1) - f(0)| \le 2 \Rightarrow |a - b| \le 2 \Rightarrow (a - b)^2 \le 4$

Now, $4a^2 + 3b^2 = 2(a + b)^2 + 2(a - b)^2 - b^2 \le 16$

$(4a^2 + 3b^2)_{\max} = 16$, when $b = 0$

$\Rightarrow |a + b| = |a - b| = |a| = 2$

Also, $\dfrac{8}{3}a^2 + 2b^2 = \dfrac{2}{3}(4a^2 + 3b^2) \le \dfrac{2}{3} \times 16 \le \dfrac{32}{3}$

21. (c) **22. (c)** **23. (b)**

24. (a) : $\dfrac{b}{2} = \lim\limits_{x \to 0} \dfrac{a\cos ax - 1 + \tan x}{2x}$

$\Rightarrow\ b = \lim\limits_{x \to 0} \dfrac{a\cos ax - 1}{x} + 1 \Rightarrow a = b = 1$

25. (c) : $\lim\limits_{x \to 0} \dfrac{1 + a\cos x - ax\sin x - b\cos x}{3x^2} = 1$

implies $a - b = -1$ and in this case,

$$1 = \lim_{x \to 0} \frac{(1 - \cos x) - ax\sin x}{3x^2} = \frac{1}{3}\left(\frac{1}{2} - a\right)$$

This gives $a = -\dfrac{5}{2}$ and $b = a + 1 = -\dfrac{3}{2}$

26. (c) : $L = \lim\limits_{x \to 0} \dfrac{\dfrac{x}{\sqrt{a^2 - x^2}} - x}{4x^3} = \lim\limits_{x \to 0} \dfrac{-1 + \dfrac{1}{\sqrt{a^2 - x^2}}}{4x^2}$

$\Rightarrow\ a = 1$ and $L = \lim\limits_{x \to 0} \dfrac{-\sqrt{1 - x^2} + 1}{4x^2\sqrt{1 - x^2}} = \dfrac{1}{8}$

(27 -28) :

We have, $\lim\limits_{n \to \infty} f(x)$

$$= \lim_{n \to \infty} \frac{n(n-1)(n-2)....(n-x+1)}{x!}\left(\frac{\lambda}{n}\right)^x \left(1 - \frac{\lambda}{n}\right)^{n-x}$$

$$= \frac{e^{-\lambda}\lambda^x}{x!}$$

$$\therefore \sum_{x=0}^{\infty} xf(x) = \sum_{x=0}^{\infty} x \cdot e^{-\lambda}\frac{\lambda^{x-1}\lambda}{(x-1)!\,x}$$

$$= \lambda e^{-\lambda} \sum_{x=1}^{\infty} \frac{\lambda^{x-1}}{(x-1)!} = e^{-\lambda}e^{\lambda}\lambda = \lambda$$

Similarly, $\sum\limits_{x=0}^{\infty} x^2 f(x) = \sum\limits_{x=0}^{\infty} x(x-1)f(x) + \sum\limits_{x=0}^{\infty} xf(x)$

$= \lambda^2 + \lambda$

27. (a) **28. (b)**

29. A → S; B → P; C → Q; D → R

A. $\dfrac{z_3 - z_1}{z_2 - z_1} = \dfrac{z_1 - z_2}{z_3 - z_2} = e^{i\frac{\pi}{3}}$

B. $\text{Re}\left(\dfrac{z_3 - z_1}{z_3 - z_2}\right) = 0 \Rightarrow \arg\left(\dfrac{z_3 - z_1}{z_3 - z_2}\right) = \pm\dfrac{\pi}{2}$

C. $\text{Re}\left(\dfrac{z_3 - z_1}{z_3 - z_2}\right) < 0 \Rightarrow \arg\left(\dfrac{z_3 - z_1}{z_3 - z_2}\right) > \dfrac{\pi}{2}$

D. $\left(\dfrac{z_3 - z_1}{z_3 - z_2}\right) = i = e^{i\frac{\pi}{2}}$ and $|z_3 - z_1| = |z_3 - z_2|$

30. A → Q, B → S, C → Q, D → Q, E → R, F → P

A. Here, $x + \dfrac{1}{x} = \sqrt{3}$

$\Rightarrow x^2 - \sqrt{3}x + 1 = 0 \Rightarrow x = \dfrac{\sqrt{3} \pm \sqrt{3 - 4}}{2}$

$\Rightarrow x = \dfrac{\sqrt{3}}{2} \pm i\left(\dfrac{1}{2}\right)$

$\Rightarrow x = \dfrac{\sqrt{3}}{2} + i\left(\dfrac{1}{2}\right)$ and $x = \dfrac{\sqrt{3}}{2} - i\left(\dfrac{1}{2}\right)$

When $x = \dfrac{\sqrt{3}}{2} + \dfrac{i}{2} \Rightarrow x = \cos\dfrac{\pi}{6} + i\sin\dfrac{\pi}{6}$

$\Rightarrow x^{2018} = \cos\dfrac{2018\pi}{6} + i\sin\dfrac{2018\pi}{6}$

$\Rightarrow \dfrac{1}{x^{2018}} = \cos\dfrac{2018\pi}{6} - i\sin\dfrac{2018\pi}{6}$

$\therefore\ x^{2018} + \dfrac{1}{x^{2018}} = 2\cos 2018\dfrac{\pi}{6} = 2\cos\left(336\pi + \dfrac{2\pi}{6}\right)$

$= 2\cos\left(336\pi + \dfrac{\pi}{3}\right) = 2\cos\left(2\pi(168) + \dfrac{\pi}{3}\right)$

$= 2\cos\dfrac{\pi}{3} = 1$

When, $x = \frac{\sqrt{3}}{2} - \frac{i}{2} = \frac{\sqrt{3}}{2} + \left(-\frac{1}{2}\right)i$

$\Rightarrow x = \cos\left(-\frac{\pi}{6}\right) + i\sin\left(-\frac{\pi}{6}\right)$

$\Rightarrow x^{2018} = \cos\frac{2018\pi}{6} + i\sin\left(\frac{-2018\pi}{6}\right)$

$\Rightarrow \frac{1}{x^{2018}} = \cos\frac{2018\pi}{6} - i\sin\left(\frac{-2018\pi}{6}\right)$

$\therefore x^{2018} + \frac{1}{x^{2018}} = 2\cos\frac{2018\pi}{6} = 2\cos\frac{\pi}{3} = 1$

B. $z = 1 + i\sqrt{3} = 2\left(\frac{1}{2} + \frac{i\sqrt{3}}{2}\right) = -2\omega^2$ and $\overline{z} = -2\omega$

$\therefore (z)^6 + (\overline{z})^6 = 2^6 + 2^6$

$\therefore \frac{(z)^6 + (\overline{z})^6}{2^5} = \frac{128}{32} = 4$

C. Since, $1, \alpha_1, \alpha_2, ..., \alpha_{n-1}$ are the n nth roots of unity, it means these are roots of the equation $x^n - 1 = 0$

Now, $x^n - 1 = (x - 1)(x - \alpha_1) \dots (x - \alpha_{n-1})$

$\Rightarrow \frac{x^n - 1}{x - 1} = (x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_{n-1})$

$\Rightarrow 1 + x^1 + x^2 + \dots + x^{n-1} = (x - \alpha_1)(x - \alpha_2) \dots (x - \alpha_{n-1})$

Putting $x = -1$ on both sides, we get

$1 + (-1)^1 + (-1)^2 + \dots (-1)^{n-1}$

$= (-1)^{n-1}(1 + \alpha_1)(1 + \alpha_2) \dots (1 + \alpha_{n-1})$ ...(*)

$\therefore (1 + \alpha_1)(1 + \alpha_2) \dots (1 + \alpha_{n-1}) = 1$

(as in the left of (*) there are n terms and n is odd)

D. Given, $z - \frac{1}{z} = i \Rightarrow z^2 - iz - 1 = 0 \Rightarrow z = \frac{i \pm \sqrt{3}}{2}$

$\Rightarrow z = \frac{i + \sqrt{3}}{2}$ and $z = \frac{i - \sqrt{3}}{2}$

$\therefore z = e^{\frac{i\pi}{6}}$ and $z = e^{i\frac{5\pi}{6}}$

$\therefore z^{2018} = e^{i2018\frac{\pi}{6}}$ and $z^{-2018} = e^{-i2018\frac{\pi}{6}}$

$\therefore z^{2018} + \frac{1}{z^{2018}} = 2\cos\frac{2018\pi}{6}$

$= 2\cos\left(336\pi + \frac{2\pi}{6}\right) = 2\cos\frac{\pi}{3} = 1$

Also, $z^{2018} = e^{i\left(\frac{2018 \times 5\pi}{6}\right)} = e^{i\left(1680\pi + \frac{10\pi}{6}\right)}$

$\therefore z^{2018} = e^{i\frac{10\pi}{6}} = e^{i\frac{5\pi}{3}}$

$\therefore z^{2018} + \frac{1}{z^{2018}} = 2\cos\frac{5\pi}{3} = 2\left(\frac{1}{2}\right) = 1$

E. Let $\alpha = |z + 3| = |(z + 4i) + (3 - 4i)|$

$\geq ||3 - 4i| - |z + 4i|| \geq 5 - 2 = 3$

F. Here, $x = -2 - i\sqrt{3}$

$\Rightarrow x + 2 = -i\sqrt{3}$

$\Rightarrow (x + 2)^2 = -3$

$\Rightarrow x^2 + 4x + 7 = 0$

Now, $2x^4 + 5x^3 + 7x^2 - x + 37$

$= 2x^2(x^2 + 4x + 7) - 3x(x^2 + 4x + 7) + 5(x^2 + 4x + 7) + 2 = 2$

$(\because x^2 + 4x + 7 = 0)$

31. A → T; B → T; C → Q; D → Q

A. Given $f(0) = 2$, $f(1) = 3$

and $f(x + 2) = 2f(x) - f(x + 1)$

Put $x = 0$; $f(2) = 2f(0) - f(1) = 2 \times 2 - 3 = 1$

Put $x = 1$; $f(3) = 2f(1) - f(2) = 2 \times 3 - 1 = 5$

Put $x = 2$; $f(4) = 2f(2) - f(3) = 2 \times 1 - 5 = -3$

Put $x = 3$; $f(5) = 2f(3) - f(4) = 2 \times 5 - (-3) = 13$

B. $\sqrt{13} > 0 \Rightarrow f\left(\sqrt{13}\right) = \left(\sqrt{13}\right)^2 = 13$

C. Given $f(x) + 2f(1 - x) = x^2 + 2, \forall x \in R$ ...(i)

Replace x by $1 - x$, we get

$f(1 - x) + 2f(x) = (1 - x)^2 + 2$ ...(ii)

Solving (i) & (ii), we get $f(x) = \frac{(x - 2)^2}{3}$

$\therefore f(5) = 3$

D. Given, $f(x) = \frac{4^x}{4^x + 2}$

$\Rightarrow f(1 - x) = \frac{4^{1-x}}{4^{1-x} + 2} = \frac{\frac{4}{4^x}}{\frac{4}{4^x} + 2} = \frac{4}{4 + 2 \cdot 4^x} = \frac{2}{4^x + 2}$

$\therefore f(x) + f(1 - x) = 1$

$\Rightarrow f\left(\frac{1}{7}\right) + f\left(\frac{6}{7}\right) = 1;\ f\left(\frac{2}{7}\right) + f\left(\frac{5}{7}\right) = 1$

and $f\left(\frac{3}{7}\right) + f\left(\frac{4}{7}\right) = 1 \Rightarrow \sum_{k=1}^{6} f\left(\frac{k}{7}\right) = 1 + 1 + 1 = 3$

32. (6) : We find that $\cos 40° + \cos 80° + \cos 160°$

$= 2\cos 60° \cos 20° - \cos 20° = 0$ ...(i)

MONTHLY TEST DRIVE CLASS XI ANSWER KEY

1. (a) **2.** (d) **3.** (d) **4.** (c) **5.** (b)
6. (b) **7.** (a,c) **8.** (b,c) **9.** (a,b,c,d)
10. (a,b,c,d) **11.** (a,b,d) **12.** (a,b,d)
13. (d) **14.** (a) **15.** (c) **16.** (a) **17.** (4)
18. (28) **19.** (3) **20.** (8)

Now, cos40°cos80° + cos80°cos160° + cos160°cos40°

$$=\frac{1}{2}(2\cos 40°\cos 80°+2\cos 80°\cos 160°+2\cos 160°\cos 40°)$$

$$=\frac{1}{2}(\cos 120°+\cos 40°+\cos 240°+\cos 80°+\cos 200°+\cos 120°)$$

$$=\frac{1}{2}\left(-\frac{1}{2}+\cos 40°-\frac{1}{2}+\cos 80°+\cos 200°-\frac{1}{2}\right)$$

$$=\frac{1}{2}\left(-\frac{3}{2}+2\cos 60°\cos 20°-\cos 20°\right)=-\frac{3}{4}\quad\text{[Using (i)]}$$

Also, cos40°cos80°cos160°

$$=\frac{\sin(2^3\times 40°)}{2^3\sin 40°}=\frac{\sin 320°}{8(\sin 40°)}=-\frac{1}{8}$$

$$\left[\because\ \cos\alpha\cos 2\alpha\cos 4\alpha\ldots\cos(2^{n-1}\alpha)=\frac{\sin 2^n\alpha}{2^n\sin\alpha}\ \text{for all } n\in N\right]$$

So, the equation having cos40°, cos80° and cos160° as its roots is given by

$$x^3-x^2\times 0+x\times\left(-\frac{3}{4}\right)-\left(-\frac{1}{8}\right)=0\ \text{ or }\ 8x^3-6x+1=0$$

The equation having sec40°, sec80° and sec160° as its roots is $x^3-6x^2+8=0$

∴ sec40° + sec80° + sec160° = 6

33. (3) : Let $f(y)=y^2-my+1$. Since 1 lies between the roots of the equation $y^2-my+1=0$

∴ $m^2-4>0$ and $1-m+1<0$

[∵ Disc > 0 and $f(1)<0$]

⇒ ($m<-2$ or $m>2$) and $m>2$ ⇒ $m>2$

Now, using A.M. – G.M. inequality, we get

$$\frac{|x|^2+16}{2}\ge 4|x|\ \Rightarrow\ 0\le\frac{4|x|}{|x|^2+16}\le\frac{1}{2}$$

$$\Rightarrow\ 0<\left(\frac{4|x|}{|x|^2+16}\right)^m<1\ \Rightarrow\ \left[\left(\frac{4|x|}{|x|^2+16}\right)^m\right]=0.$$

34. (5) : We have, $x=4t^3+3$ and $y=4+3t^4$

$$\therefore\ \frac{dx}{dt}=12t^2\ \text{and}\ \frac{dy}{dt}=12t^3$$

$$\Rightarrow\ \frac{dy}{dx}=\frac{12t^3}{12t^2}=t\quad\therefore\ \frac{dx}{dy}=\frac{1}{t}=t^{-1}$$

$$\Rightarrow\ \frac{d^2x}{dy^2}=-t^{-2}\frac{dt}{dy}=-t^{-2}\times\frac{1}{12t^3}=-\frac{1}{12}t^{-5}$$

$$\therefore\ \frac{d^2x}{dy^2}\Bigg/\left(\frac{dx}{dy}\right)^n=-\frac{1}{12}t^{-5}\times t^n=-\frac{1}{12}t^{n-5}$$

Since $\frac{d^2x}{dy^2}\Big/\left(\frac{dx}{dy}\right)^n$ is constant. Therefore, $n=5$.

35. (5) : For infinite tangent it should be a point circle, so radius = 0

$$\Rightarrow\ \sqrt{4+1-\lambda}=0$$

⇒ $\lambda=5$.

36. (3) : Given $\begin{vmatrix} x^3+1 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix}=30$

$$\Rightarrow\ \begin{vmatrix} x^3 & x^2y & x^2z \\ xy^2 & y^3+1 & y^2z \\ xz^2 & yz^2 & z^3+1 \end{vmatrix}+\begin{vmatrix} 1 & x^2y & x^2z \\ 0 & y^3+1 & y^2z \\ 0 & yz^2 & z^3+1 \end{vmatrix}=30$$

$$\Rightarrow\ x\begin{vmatrix} x^2 & x^2y & x^2z \\ y^2 & y^3+1 & y^2z \\ z^2 & yz^2 & z^3+1 \end{vmatrix}+(y^3+1)(z^3+1)-y^3z^3=30$$

Applying $C_2\to C_2-yC_1$ and $C_3\to C_3-zC_1$, we get

$$x\begin{vmatrix} x^2 & 0 & 0 \\ y^2 & 1 & 0 \\ z^2 & 0 & 1 \end{vmatrix}+y^3z^3+z^3+y^3+1-y^3z^3=30$$

⇒ $x^3+y^3+z^3+1=30$ ⇒ $x^3+y^3+z^3=29$

As $29=3^3+1^3+1^3$, then solutions are (3, 1, 1) (1, 3, 1) and (1, 1, 3).

Hence there are three positive integral solutions.

37. (0) : Given, $3\vec{a}+4\vec{b}+5\vec{c}=0$

$$\Rightarrow\ 3\vec{a}+4\vec{b}=-5\vec{c}$$

$$\Rightarrow\ 9+16+24\vec{a}\cdot\vec{b}=25\qquad\text{(Squaring both sides)}$$

$$\Rightarrow\ \vec{a}\cdot\vec{b}=0$$

38. (0) : Since α, β, γ are the roots of $x^3+px+q=0$

∴ $\alpha+\beta+\gamma=0$

Applying $C_1\to C_1+C_2+C_3$, then we get

$$\begin{vmatrix} \alpha+\beta+\gamma & \beta & \gamma \\ \alpha+\beta+\gamma & \gamma & \alpha \\ \alpha+\beta+\gamma & \alpha & \beta \end{vmatrix}=\begin{vmatrix} 0 & \beta & \gamma \\ 0 & \gamma & \alpha \\ 0 & \alpha & \beta \end{vmatrix}=0$$

39. (5) : Let $t_r = \frac{{}^nC_{r-1}}{{}^nC_r + {}^nC_{r-1}} = \frac{{}^nC_{r-1}}{{}^{n+1}C_r} = \frac{{}^nC_{r-1}}{\frac{n+1}{r}\,{}^nC_{r-1}}$

$\therefore \quad t_r = \frac{r}{n+1}$

Now, $S = \sum_{r=0}^{n} \{t_r\}^3 = \sum_{r=0}^{n} \frac{r^3}{(n+1)^3} = \frac{1}{(n+1)^3}\sum_{r=0}^{n} r^3$

$\Rightarrow \; S = \frac{1}{(n+1)^3}\left\{\frac{n(n+1)}{2}\right\}^2 = \frac{n^2}{4(n+1)}$

Now, $S = \frac{25}{24}$ (given) which is only possible for $n = 5$.

40. (65) : Let $u_k = \frac{k+2}{k(k+1)(k+3)}$

$$= \frac{(k+2)^2}{k(k+1)(k+2)(k+3)}$$

$$= \frac{k^2+4k+4}{k(k+1)(k+2)(k+3)} = \frac{k(k+1)+3k+4}{k(k+1)(k+2)(k+3)}$$

$$= \frac{1}{(k+2)(k+3)} + \frac{3}{(k+1)(k+2)(k+3)} + \frac{4}{k(k+1)(k+2)(k+3)}$$

$$= \left(\frac{1}{k+2} - \frac{1}{k+3}\right) - \frac{3}{2}\left[\frac{1}{(k+2)(k+3)} - \frac{1}{(k+1)(k+2)}\right]$$

$$-\frac{4}{3}\left[\frac{1}{(k+1)(k+2)(k+3)} - \frac{1}{k(k+1)(k+2)}\right]$$

Now, put $k = 1, 2, 3, \ldots., n$ and add.

Thus, $s_n = u_1 + u_2 + \ldots\ldots + u_n$

$$= \left(\frac{1}{3} - \frac{1}{n+3}\right) - \frac{3}{2}\left[\frac{1}{(n+2)(n+3)} - \frac{1}{2\cdot 3}\right]$$

$$-\frac{4}{3}\left[\frac{1}{(n+1)(n+2)(n+3)} - \frac{1}{1\cdot 2\cdot 3}\right]$$

Therefore $\lim_{n\to\infty} s_n = \frac{1}{3} + \frac{3}{12} + \frac{4}{18} = \frac{29}{36} = \frac{a}{b}$

$\therefore \quad a + b = 29 + 36 = 65$

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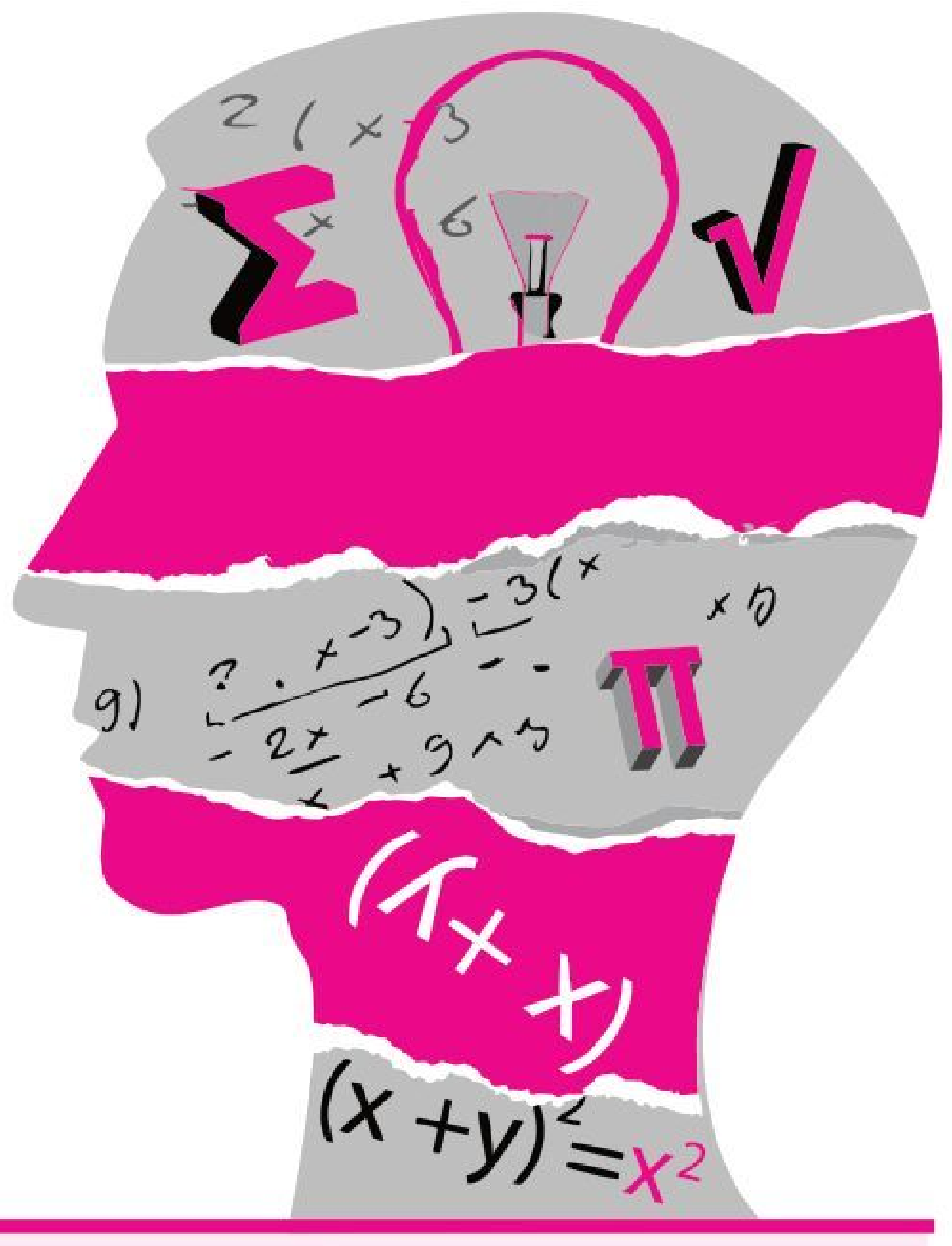

Math Archives, as the title itself suggests, is a collection of various challenging problems related to the topics of JEE (Main & Advanced) Syllabus. This section is basically aimed at providing an extra insight and knowledge to the candidates preparing for JEE (Main & Advanced). In every issue of MT, challenging problems are offered with detailed solution. The reader's comments and suggestions regarding the problems and solutions offered are always welcome.

1. Equation of a common tangent to the curves $y^2 = 8x$ and $xy = -1$ is

(a) $3y = 9x + 2$ (b) $y = 2x + 1$

(c) $2y = x + 8$ (d) $y = x + 2$

2. Integrating factor of the differential equation $2y\cos y^2 \frac{dy}{dx} - \frac{2}{x+1}\sin y^2 = (x+1)^3$ is

(a) $(x+1)^2$ (b) $\log_e(x+1)$

(c) $e^{\log_e(x+1)}$ (d) $\frac{1}{(x+1)^2}$

3. The radius of a right circular cylinder of maximum volume which can be inscribed in a sphere of radius R is

(a) R (b) $\frac{R}{2}$ (c) $\sqrt{\frac{2}{3}}R$ (d) $\sqrt{\frac{3}{2}}R$

4. The distance between the line $\frac{x-2}{1} = \frac{y+2}{-1} = \frac{z-3}{4}$ and the plane $x + 5y + z = 5$ is

(a) $\frac{10}{3\sqrt{3}}$ units (b) $\frac{10}{3}$ units

(c) $\frac{10}{9}$ units (d) $\frac{10}{7}$ units

5. The number of ways in which 13 identical gold coins can be distributed among three persons such that each person can get at least two gold coins is

(a) 36 (b) 24

(c) 12 (d) 6

6. If A, B, C are the angles of a triangle, the system of equations $(\sin A)x + y + z = \cos A$, $x + (\sin B)y + z = \cos B$, $x + y + (\sin C)z = 1 - \cos C$ has

(a) no solution

(b) unique solution

(c) infinitely many solutions

(d) finitely many solutions

7. Let $f(x) = 2x^n + \lambda$, $\lambda \in R$, $f(4) = 133$ and $f(5) = 255$ then sum of the factors of $(f(3) - f(2))$ is

(a) 20 (b) 60 (c) 21 (d) 59

8. If $|z^2 - 1| = |z|^2 + 1$, then z lies on

(a) the real axis (b) an ellipse

(c) a circle (d) the imaginary axis

9. If $x^2 - 4x + 5 - \sin y = 0$, $y \in [0, 2\pi]$, then

(a) $x = 1, y = 0$ (b) $x = 1, y = \pi/2$

(c) $x = 2, y = 0$ (d) $x = 2, y = \pi/2$

10. A solution of the equation $\tan^{-1}(1+x) - \tan^{-1}(x-1) = \frac{\pi}{2}$ is

(a) $x = 1$ (b) $x = -1$ (c) $x = 0$ (d) $x = \pi$

11. Let (x_0, y_0) be the solution of the following equations $(2x)^{\ln 2} = (3y)^{\ln 3}$ and $3^{\ln x} = 2^{\ln y}$. Then x_0 is

(a) $\frac{1}{6}$ (b) $\frac{1}{3}$ (c) $\frac{1}{2}$ (d) 6

12. Let $S(K) = 1 + 3 + 5 + \ldots\ldots + (2K - 1) = 3 + K^2$. Then which of the following is true ?

(a) $S(1)$ is correct

(b) Principle of mathematical induction can be used to prove the formula

(c) $S(K) \nRightarrow S(K+1)$
(d) $S(K) \Rightarrow S(K+1)$

13. The real part of $(1 - \cos\theta + 2i\sin\theta)^{-1}$ is

(a) $\dfrac{1}{3+5\cos\theta}$ (b) $\dfrac{1}{5-3\cos\theta}$
(c) $\dfrac{1}{3-5\cos\theta}$ (d) $\dfrac{1}{5+3\cos\theta}$

14. a, g, h are arithmetic mean, geometric mean and harmonic mean between two positive numbers x and y respectively. Then identify the correct statement among the following.

(a) h is the harmonic mean between a and g
(b) No such relation exists between a, g and h
(c) g is the geometric mean between a and h
(d) a is the arithmetic mean between g and h

15. If $s_r = \alpha^r + \beta^r + \gamma^r$, then the value of $\begin{vmatrix} s_0 & s_1 & s_2 \\ s_1 & s_2 & s_3 \\ s_2 & s_3 & s_4 \end{vmatrix}$ is equal to

(a) 0 (b) $(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$
(c) $(\alpha+\beta+\gamma)^6$ (d) $(\alpha-\beta)^2(\beta-\gamma)^2(\gamma-\alpha)^2$

16. p : Sita is beautiful; q = Sita is clever.
When $\sim p \vee q$ is written in verbal form it becomes

(a) Sita is not beautiful or Sita is not clever
(b) Sita is beautiful and she is clever.
(c) Sita is not beautiful or Sita is clever
(d) Sita is beautiful and Sita is not clever

17. Ten persons, amongst whom are A, B and C are to speak at a function. The number of ways in which it can be done if A wants to speak before B and B wants to speak before C is

(a) $\dfrac{10!}{6}$ (b) 3! 7!
(c) ${}^{10}P_3 \cdot 7!$ (d) None of these

18. If the coefficient of x^7 in $\left(ax^2 + \dfrac{1}{bx}\right)^{11}$ is equal to the coefficient of x^{-7} in $\left(ax - \dfrac{1}{bx^2}\right)^{11}$, then ab =

(a) 1 (b) 1/2 (c) 2 (d) 3

19. If the probability density function of a random variable X is $f(x) = x/2$ in $0 \le x \le 2$, then $P(X > 1.5 \mid X > 1)$ is equal to

(a) 7/16 (b) 3/4
(c) 7/12 (d) 21/64

20. The product of lengths of perpendiculars drawn from the origin to the lines represented by the equation, $ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$, will be

(a) $\dfrac{ab}{\sqrt{a^2-b^2+4h^2}}$ (b) $\dfrac{bc}{\sqrt{a^2-b^2+4h^2}}$
(c) $\dfrac{ca}{\sqrt{(a^2+b^2)+4h^2}}$ (d) $\dfrac{c}{\sqrt{(a-b)^2+4h^2}}$

21. If two distinct chords, drawn from the point (p, q) on the circle $x^2 + y^2 = px + qy$, (where $pq \ne 0$) are bisected by the x-axis, then

(a) $p^2 = q^2$ (b) $p^2 = 8q^2$
(c) $p^2 < 8q^2$ (d) $p^2 > 8q^2$

22. A circle C_1 of radius 2 touches both x-axis and y-axis. Another circle C_2 whose radius is greater than 2 touches circle C_1 and both the axes. Then the radius of circle C_2 is

SAMURAI SUDOKU

Samurai Sudoku puzzle consists of five overlapping sudoku grids. The standard sudoku rules apply to each 9 × 9 grid. Place digits from 1 to 9 in each empty cell. Every row, every column and every 3 × 3 box should contain one of each digit.

The puzzle has a unique solution.

		1	3		9	8							2	6			8	5		
	3					2	6	9					3		5	2			4	
8			2			3	1					8				9		7	2	3
6	4		7					1				3		7			2			6
3	1			9			4							8			6		5	2
	9	8			1		3					2		5	3				8	
		3	1		8	4		7	3	5		6					5		3	
		4		3				6	9				8		2	6			7	
9				2		1				2			7				3	2		
						8	3			9			5							
							7	9			5		2							
						2			8	4		3								
4			7			9	1		5		4	7			9		2		6	
	8	9	4					5		8	9	2				3	6		7	
		1		9	5	7					3			5	7		8			3
	4			1		8	9					8		3		6				7
	9				8		7	4				6	7				9	2		
8			9			1							2			7	3			6
		7	8			4								7		2		6		
5				4	9								8		6				9	
2			1					9						6			7	3		

Readers can send their responses at editor@mtg.in or post us with complete address. Winners' name with their valuable feedback will be published in next issue.

(a) $6-4\sqrt{2}$ (b) $6+4\sqrt{2}$

(c) $6-4\sqrt{3}$ (d) $6+4\sqrt{3}$

23. If $\tan\theta=\dfrac{\sin\alpha-\cos\alpha}{\sin\alpha+\cos\alpha}$, then $\sin\alpha$ + $\cos\alpha$ and $\sin\alpha$ – $\cos\alpha$ must be respectively equal to

(a) $\sqrt{2}\cos\theta, \sqrt{2}\sin\theta$ (b) $\sqrt{2}\sin\theta, \sqrt{2}\cos\theta$

(c) $\sqrt{2}\sin\theta, \sqrt{2}\sin\theta$ (d) $\sqrt{2}\cos\theta, \sqrt{2}\cos\theta$

24. A pair of straight lines drawn through the origin form with the line $2x + 3y = 6$ an isosceles right angled triangle, then the area of the triangle thus formed is

(a) $\Delta=\dfrac{36}{13}$ (b) $\Delta=\dfrac{12}{17}$

(c) $\Delta=\dfrac{13}{5}$ (d) None of these

25. Point D, E are taken on the side BC of a triangle ABC such that $BD = DE = EC$. If $\angle BAD = x$, $\angle DAE = y$, $\angle EAC = z$, then the value of $\dfrac{\sin(x+y)\sin(y+z)}{\sin x\sin z}=$

(a) 1 (b) 2

(c) 4 (d) None of these

26. If $\tan^{-1}x + \tan^{-1}y + \tan^{-1}z = \pi$, then $x + y + z$ is equal to

(a) xyz (b) 0

(c) 1 (d) $2xyz$

27. From the bottom of a pole of height h, the angle of elevation of the top of a tower is α and the pole subtends angle β at the top of the tower. The height of the tower is

(a) $\dfrac{h\tan(\alpha-\beta)}{\tan(\alpha-\beta)-\tan\alpha}$ (b) $\dfrac{h\cot(\alpha-\beta)}{\cot(\alpha-\beta)-\cot\alpha}$

(c) $\dfrac{\cot(\alpha-\beta)}{\cot(\alpha-\beta)-\cot\alpha}$ (d) None of these

28. If f is an even function defined on the interval $(-5, 5)$, then four real values of x satisfying the equation $f(x) = f\left(\dfrac{x+1}{x+2}\right)$ are

(a) $\dfrac{-3-\sqrt{5}}{2}, \dfrac{-3+\sqrt{5}}{2}, \dfrac{3-\sqrt{5}}{2}, \dfrac{3+\sqrt{5}}{2}$

(b) $\dfrac{-5+\sqrt{3}}{2}, \dfrac{-3+\sqrt{5}}{2}, \dfrac{3+\sqrt{5}}{2}, \dfrac{3-\sqrt{5}}{2}$

(c) $\dfrac{3-\sqrt{5}}{2}, \dfrac{3+\sqrt{5}}{2}, \dfrac{-3-\sqrt{5}}{2}, \dfrac{5+\sqrt{3}}{2}$

(d) $-3-\sqrt{5}, -3+\sqrt{5}, 3-\sqrt{5}, 3+\sqrt{5}$

29. The value of $\lim\limits_{x\to\frac{\pi}{2}}\dfrac{\int_{\pi/2}^{x} t\,dt}{\sin(2x-\pi)}$ is

(a) ∞ (b) $\dfrac{\pi}{2}$

(c) $\dfrac{\pi}{4}$ (d) $\dfrac{\pi}{8}$

30. If $f'(x) = g(x)$ and $g'(x) = -f(x)$ for all x and $f(2) = 4 = f'(2)$ then $f^2(4) + g^2(4)$ is

(a) 8 (b) 16

(c) 32 (d) 64

31. The value of p for which the function,

$$f(x)=\begin{cases}\dfrac{(4^x-1)^3}{\sin\dfrac{x}{p}\log_e\left(1+\dfrac{x^2}{3}\right)}, & x\neq 0\\ 12(\log_e 4)^3, & x=0\end{cases}$$

is continuous at $x = 0$, is

(a) 1 (b) 2

(c) 3 (d) 4

32. Maximize $z = 3x + y$ subject to $x + y \le 6$, $y \le 4$, $x \ge 1$, $x \ge 0$ and $y \ge 0$.

(a) 20 (b) 18

(c) 10 (d) 15

33. Let $f(x)=\int_0^x \dfrac{\cos t}{t}\,dt$, $x>0$, then $f(x)$ has

(a) Maxima when $n = -2, -4, -6, \ldots.$

(b) Maxima when $n = -1, -3, -5, \ldots.$

(c) Maxima when $n = 1, 3, 5, \ldots.$

(d) None of these

34. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i}-\hat{j}$, $4\hat{i}$, $3\hat{i}+3\hat{j}$ and $-3\hat{i}+2\hat{j}$ respectively. The quadrilateral $PQRS$ must be a

(a) Parallelogram, which is neither a rhombus nor a rectangle

(b) Square

(c) Rectangle, but not a square

(d) Rhombus, but not a square

35. Let $A = \{2, 3, 4, 5\}$ and let $R\{(2, 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)\}$ be a relation in A. Then R is

(a) reflexive and transitive

(b) reflexive and symmetric

(c) transitive and symmetric

(d) equivalence

36. The value of 'a' so that the volume of parallelepiped formed by $\hat{i} + a\hat{j} + \hat{k}$, $\hat{j} + a\hat{k}$ and $a\hat{i} + \hat{k}$ becomes minimum, is

(a) -3 (b) 3

(c) $\dfrac{1}{\sqrt{3}}$ (d) $\sqrt{3}$

37. $$\int \frac{dx}{(1+x^2)\sqrt{p^2+q^2(\tan^{-1}x)^2}} =$$

(a) $\dfrac{1}{q}\log\left[q\tan^{-1}x + \sqrt{p^2+q^2(\tan^{-1}x)^2}\right] + c$

(b) $\log\left[q\tan^{-1}x + \sqrt{p^2+q^2(\tan^{-1}x)^2}\right] + c$

(c) $\dfrac{2}{3q}(p^2+q^2\tan^{-1}x)^{3/2} + c$

(d) None of these

38. Mean of 100 items is 49. It was discovered that three items which should have been 60, 70, 80 were wrongly read as 40, 20, 50 respectively. The correct mean is

(a) 80 (b) 50

(c) $82\frac{1}{2}$ (d) 48

39. A tangent to an ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ meets the co-ordinate axes in P and Q. The least value of PQ is

(a) $a - b$ (b) $a + b$

(c) $\dfrac{a+b}{2}$ (d) $2a$

40. The plane passing through the point $(-2, -2, 2)$ and containing the line joining the points $(1, 1, 1)$ and $(1, -1, 2)$ makes intercepts on the co-ordinate axes, the sum of length of intercepts is

(a) 3 (b) 4

(c) 6 (d) 12

ANSWER KEY

1.	(d)	**2.**	(d)	**3.**	(c)	**4.**	(a)	**5.**	(a)
6.	(b)	**7.**	(b)	**8.**	(d)	**9.**	(d)	**10.**	(c)
11.	(c)	**12.**	(d)	**13.**	(d)	**14.**	(c)	**15.**	(d)
16.	(c)	**17.**	(a)	**18.**	(a)	**19.**	(c)	**20.**	(d)
21.	(d)	**22.**	(b)	**23.**	(a)	**24.**	(a)	**25.**	(c)
26.	(a)	**27.**	(b)	**28.**	(a)	**29.**	(c)	**30.**	(c)
31.	(d)	**32.**	(b)	**33.**	(b)	**34.**	(a)	**35.**	(b)
36.	(c)	**37.**	(a)	**38.**	(b)	**39.**	(b)	**40.**	(d)

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CBSE warm-up!

CLASS-XI

TERM-I OBJECTIVE TYPE QUESTIONS*

Series 4 Sequences and Series

Time Allowed : 90 min.
Maximum Marks : 40

General Instructions :

1. *This question paper contains three sections A, B and C. Each part is compulsory.*
2. *Section - A has 20 MCQs, attempt any 16 out of 20.*
3. *Section - B has 20 MCQs, attempt any 16 out of 20*
4. *Section - C has 10 MCQs, attempt any 8 out of 10.*
5. *There is no negative marking.*
6. *All questions carry equal marks.*

SECTION-A

In this section, attempt any 16 questions out of Questions 1-20. Each Question is of 1 mark weightage.

1 What is the 20th term of the sequence defined by $a_n = (n - 1)(2 - n)(3 + n)$?
(a) 7866 (b) −7866 (c) 4598 (d) −4598

2. Let the sequence a_n be defined as follows : $a_1 = 1, a_n = a_{n-1} + 2$ for $n \geq 2$.
Write the corresponding series.
(a) $1 + 3 + 5 + 7 + 9 + ...$
(b) $1 + 5 + 9 + 13 + 17 + ...$
(c) $1 + 4 + 7 + 10 + 13 + ...$
(d) $1 + 6 + 11 + 16 + 21 + ...$

3. In an A.P., if m^{th} term is n and the n^{th} term is m, where $m \neq n$, then find its p^{th} term.
(a) $n - m + p$ (b) $n + m + p$
(c) $n + m - p$ (d) $n - m - p$

4. The income of a person is ₹ 300000, in the first year and he receives an increase of ₹ 10000 to his income per year for the next 19 years. Find the total amount, he received in 20 years.
(a) ₹ 7800000 (b) ₹ 7900000
(c) ₹ 7700000 (d) ₹ 790000

5. Which term of the sequence 4, 9, 14, 19, ... is 124?
(a) 25th (b) 20th (c) 26th (d) 21st

6. Find the sum of all odd integers between 2 and 100 divisible by 3.
(a) 862 (b) 863
(c) 865 (d) 867

7. Let the positive numbers a, b, c, d be in A.P. Then
(a) $\frac{1}{abc}, \frac{1}{abd}, \frac{1}{acd}, \frac{1}{bcd}$ are in A.P.
(b) $\frac{1}{abc}, \frac{1}{acd}, \frac{1}{bcd}, \frac{1}{abd}$ are in A.P.

**Chapterwise practice questions for CBSE Exam Term- I as per the pattern issued by CBSE.*

(c) $\frac{1}{acd}, \frac{1}{bcd}, \frac{1}{abc}, \frac{1}{abd}$ are in A.P.

(d) none of these

8. If S_1, S_2, S_3 are the sum of $n, 2n, 3n$ terms respectively of an A.P., then $S_3/(S_2 - S_1) =$

(a) 1 (b) 2 (c) 3 (d) 4

9. If 1, $\log_{81}(3^x + 48)$ and $\log_9\left(3^x - \frac{8}{3}\right)$ are in A.P., then x is equal to

(a) 1 (b) 2 (c) 9 (d) 3

10. There are m A.M.'s between 1 and 31. If the ratio of the 7th and $(m - 1)^{\text{th}}$ means is 5 : 9, then $m =$

(a) 10 (b) 12 (c) 14 (d) 16

11. The product of three numbers in an A.P. is 224, and the largest number is 7 times the smallest. Find the numbers.

(a) 1, 5, 7 (b) 2, 6, 8

(c) 2, 8, 14 (d) 4, 8, 10

12. The 6th term of an A.P. is 18 and the 9th term is 12. The 15th term is equal to

(a) 0 (b) 1 (c) 2 (d) 3

13. If sum to n terms of an A.P. is $5n^2 + 6n$ and r^{th} term is 401, then the value of r is

(a) 40 (b) 30 (c) 20 (d) 10

14. If log2, $\log(2^x - 1)$ and $\log(2^x + 3)$ are in A.P., then the value of x is

(a) 5/2 (b) $\log_2 5$

(c) $\log_3 5$ (d) $\log_5 3$

15. The third term of a G.P. is 42. Find the product of its first five terms.

(a) 42 (b) $(42)^5$ (c) 98 (d) $(25)^5$

16. Find the 9th term and the general term of the progression $\frac{1}{4}, -\frac{1}{2}, 1, -2, \ldots$

(a) 64, $(-1)^{n-1}\, 2^{n-3}$ (b) 63, $(-1)^{n-1}\, 2^{n-2}$

(c) 62, $(-1)^{n-2}\, 2^{n-3}$ (d) 61, $(-2)^{n-1}\, 2^{n-3}$

17. Three numbers, the third of which being 12, form a decreasing G.P. If the third term was 9 instead of 12, the three numbers would have formed an A.P., then the common ratio of the original G.P., is

(a) $\frac{3}{7}$ (b) $\frac{2}{3}$ (c) $\frac{3}{5}$ (d) $\frac{4}{5}$

18. The A.M. of the roots of a quadratic equation is A and G.M. of its roots is G. The quadratic equation is

(a) $x^2 + Ax + G^2 = 0$ (b) $x^2 + 2Ax + G^2 = 0$

(c) $x^2 - Ax + G^2 = 0$ (d) $x^2 - 2Ax + G^2 = 0$

19. Let x be the arithmetic mean and y, z be the two geometric means between any two positive numbers. The value of $\frac{y^3 + z^3}{xyz}$ is

(a) 1 (b) 2 (c) 3 (d) 4

20. If x and y are positive real numbers and m, n are positive integers, then the maximum value of $\frac{x^m y^n}{(1 + x^{2m})(1 + y^{2n})}$ is

(a) 2 (b) $\frac{1}{4}$ (c) $\frac{1}{2}$ (d) 1

SECTION-B

In this section, attempt any 16 questions out of the Questions 21 - 40. Each Question is of 1 mark weightage.

21. Find the 440th and 441st terms of the sequence given by

$$t_n = \begin{cases} \dfrac{n}{\dfrac{n}{44} - 1}, & \text{if } n \text{ is not the square of a natural number.} \\ 2.7, & \text{if } n \text{ is the square of a natural number.} \end{cases}$$

(a) $\frac{440}{9}, 2.7$ (b) $\frac{9}{440}, 2.7$

(c) 440, 9 (d) 9, 3.7

22. If a, b, c, d, e are in A.P., then find the value of $a - 4b + 6c - 4d + e$.

(a) $3b$ (b) 0 (c) $4a$ (d) $6c$

23. Find the values of 'k' for which $-\frac{2}{7}, k, -\frac{7}{2}$ are in G.P.

(a) 0 (b) ±4

(c) ±2 (d) ±1

24. At the end of each year, the value of a certain machine has depreciated by 20% of its value at the beginning of that year. If its initial value was ₹ 1250, then find the value at the end of 5 years.

(a) ₹ 409.60 (b) ₹ 408.50

(c) ₹ 407.20 (d) ₹ 409

25. Find the sum to infinity for $5, \frac{5}{3}, \frac{5}{9}, \ldots\ldots$

(a) 7.5 (b) 15

(c) 7 (d) 17

26. If one G.M. is g and two A.M.'s are p and q, are inserted between two numbers a and b, then $\frac{(2p-q)(p-2q)}{g^2} =$

(a) 1 (b) –1 (c) 2 (d) –3

27. If a, b, c are positive numbers in A.P., such that their product is 64, then the minimum value of b is equal to

(a) 2 (b) 4 (c) 1 (d) 5

28. If a be a positive real number and A.M. of a and $2a$ exceeds the G.M. by 2, then a is equal to

(a) $3(4 \pm \sqrt{2})$ (b) $3 \pm 2\sqrt{2}$

(c) $4(3 \pm 2\sqrt{2})$ (d) $4 \pm 2\sqrt{2}$

29. If the sum of n terms of an A.P. is $3n^2 + 5n$ and its m^{th} term is 164, then find the value of m.

(a) 23 (b) 27 (c) 24 (d) 25

30. If $\log_{2^{1/2}} a + \log_{2^{1/4}} a + \log_{2^{1/6}} a + \log_{2^{1/8}} a + \ldots$ upto 20 terms is 840, then a is equal to

(a) 2 (b) 1 (c) 4 (d) $\sqrt{2}$

31. The natural numbers are grouped as follows: $S_1 = \{1\}, S_2 = \{2, 3, 4\}, S_3 = \{5, 6, 7, 8, 9\}, \ldots$, then the first element of S_{21} is

(a) 391 (b) 399 (c) 401 (d) 442

32. Find three arithmetic means between 3 and 19.

(a) 7, 11, 15 (b) 5, 10, 15

(c) 6, 11, 16 (d) 5, 7, 9

33. 28 is divided into 4 parts which are in A.P. The ratio of the product of the first and third to the product of the second and fourth is 8 : 15. The largest part is

(a) 6 (b) 8 (c) 10 (d) 12

34. In a G.P. of positive terms, if any term is equal to the sum of the next two terms. Then the common ratio of the G.P. is

(a) sin 18° (b) 2cos18°

(c) cos18° (d) 2sin18°

35. If a_1, a_2, a_3 (with $a_1 > 0$) are in G.P. with common ratio r, then the value of r with which the inequality $9a_1 + 5a_3 > 14a_2$ holds, cannot lie in the interval

(a) $\left[1, \frac{9}{2}\right]$ (b) $(-\infty, 0)$

(c) $\left[\frac{5}{9}, 1\right]$ (d) $\left[1, \frac{9}{5}\right]$

36. If a, b, c, d are in G.P., then $(b-c)^2 + (c-a)^2 + (b-d)^2 =$

(a) $(c+d)^2$ (b) $(c-d)^2$

(c) $(a+d)^2$ (d) $(a-d)^2$

37. If a, b, c, d are in G.P., then

$\frac{(a^2+b^2+c^2)(b^2+c^2+d^2)}{(ab+bc+cd)^2} =$

(a) 1 (b) 2 (c) 3 (d) 5

38. A person has 2 parents, 4 grandparents, 8 great grandparents, and so on. Find the number of his ancestors during the ten generations preceding his own.

(a) 2042 (b) 2046 (c) 2044 (d) 2045

39. $(2n + 1)$ G.M.'s are inserted between 4 and 2916. Then the $(n + 1)^{\text{th}}$ G.M. is equal to

(a) 36 (b) 54 (c) 108 (d) 324

40. If A.M. and G.M. of two positive numbers a and b are 10 and 8, respectively, then find the numbers.

(a) 4, 16 (b) 16, 4

(c) Both (a) and (b) (d) None of these

SECTION-C

- *In this section, attempt any 8 questions.*
- *Each question is of 1-mark weightage.*
- *Questions 46-50 are based on a Case-Study.*

41. The sum of the A.M. and G.M. of two positive numbers is equal to the difference between the numbers. The numbers are in the ratio

(a) 1 : 3 (b) 1 : 6 (c) 9 : 1 (d) 1 : 12

42. If g_1, g_2 are two geometric means and a_1 is the arithmetic mean between two positive numbers, then the value of $\frac{g_1^2}{g_2} + \frac{g_2^2}{g_1}$ is

(a) $2a_1$ (b) a_1 (c) $\frac{a_1}{2}$ (d) $3a_1$

43. Find the sum of the sequence 7, 77, 777, 7777, … to n terms.

(a) $\frac{7}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$ (b) $\frac{7}{9}\left[\frac{10(10^n + 1)}{9} - n\right]$

(c) $\frac{10^n - 1}{9}$ (d) None of these

44. How many terms of the geometric series $1 + 4 + 16 + 64 + \ldots$ will make the sum 5461?

(a) 6 (b) 7 (c) 8 (d) 9

45. In a G.P. of even number of terms, the sum of all terms is 5 times the sum of the odd terms. The common ratio of the G.P. is

(a) $\frac{-4}{5}$ (b) $\frac{1}{5}$ (c) 4 (d) –5

CASE STUDY

A company manufactured 1200 L.E.D. TV in 2nd year and 1400 L.E.D. TV in 6th year. Assume that the production increases uniformly by a constant number every year.

Based on the given information, answer the following questions.

46. The fixed number by which production is increasing every year, is
(a) 25 (b) 50 (c) 75 (d) 100

47 Find the production in 1st year.
(a) 1000 units (b) 1050 units
(c) 1100 units (d) 1150 units

48. Number of L.E.D. TV manufactured in 10th year, is
(a) 1600 units (b) 1650 units
(c) 1500 units (d) 1150 units

49. Total production in 10 years is
(a) 12500 units (b) 13750 units
(c) 14000 units (d) 18000 units

50. If $S_n = n^2q$, n is any natural number then $T_q =$
(a) $q(2q-1)$ (b) $q(q-1)$
(c) $q^2(q-1)$ (d) mnq

SOLUTIONS

1. **(b) :** Putting $n = 20$, we get
$a_{20} = (20-1)(2-20)(3+20) = -7866$

2. **(a) :** We have, $a_n = a_{n-1} + 2, n \geq 2$ and $a_1 = 1$
$\therefore$ $a_2 = a_1 + 2 = 1 + 2 = 3, a_3 = a_2 + 2 = 3 + 2 = 5,$
$a_4 = a_3 + 2 = 5 + 2 = 7, a_5 = a_4 + 2 = 7 + 2 = 9.$
$\therefore$ The corresponding series is $1 + 3 + 5 + 7 + 9 + \ldots$

3. **(c) :** We have, $a_m = a + (m-1)d = n$ …(i)
and $a_n = a + (n-1)d = m$ …(ii)
Solving (i) and (ii), we get
$(m-n)d = n - m \Rightarrow d = -1$
So, $a = n + m - 1$
Now, $a_p = a + (p-1)d = n + m - 1 + (p-1)(-1)$
$= n + m - p$

4. **(b) :** We have, an A.P. with $a = 300000$, $d = 10000$ and $n = 20$
$\therefore$ $S_{20} = \frac{20}{2}[600000 + 19 \times 10000] = 7900000$

5. **(a) :** Clearly, the given sequence is an A.P. with first term, $a = 4$ and common difference, $d = 5$.
Let 124 be the n^{th} term of the given sequence. Then,
$a_n = 124 \Rightarrow a + (n-1)d = 124$
$\Rightarrow$ $4 + (n-1) \times 5 = 124 \Rightarrow n = 25$
Hence, 25th term of the given sequence is 124.

6. **(d) :** The odd integers between 2 and 100 which are divisible by 3 are 3, 9, 15, 21, ..., 99. Clearly, it is an A.P. with first term, $a = 3$ and common difference, $d = 6$. Let there be n terms in this sequence. Then,
$l = a_n = 99 \Rightarrow 3 + (n-1) \times 6 = 99 \Rightarrow n = 17$
$\therefore$ Required sum $= \frac{n}{2}[a+l] = \frac{17}{2}[3+99] = 867$

7. **(a) :** a, b, c, d are in A.P.
Reversing the terms, d, c, b, a are also in A.P.
Dividing each term by $abcd$,
$\frac{1}{abc}, \frac{1}{abd}, \frac{1}{acd}, \frac{1}{bcd}$ are in A.P.

8. **(c) :** $S_1 = \frac{n}{2}[2a + (n-1)d]$
$S_2 = n[2a + (2n-1)d]$
$S_2 - S_1 = na + (3n-1)\frac{nd}{2} = \frac{n}{2}[2a + (3n-1)d]$
Now, $S_3 = \frac{3n}{2}[2a + (3n-1)d]$
$\therefore$ $S_3/(S_2 - S_1) = 3$

9. **(b) :** The three numbers $\log_9 9$, $\log_{9^2}(3^x + 48)$ and $\log_9\left(3^x - \frac{8}{3}\right)$ are in A.P.
$\Rightarrow$ $\log_9 9, \frac{1}{2}\log_9(3^x + 48)$ and $\log_9\left(3^x - \frac{8}{3}\right)$ are in A.P.
$\Rightarrow$ $3^x + 48 = 9\left(3^x - \frac{8}{3}\right) \Rightarrow 8 \times 3^x = 72$
$\Rightarrow$ $3^x = 9 \Rightarrow x = 2$

10. **(c) :** We have, $1, a_1, a_2, \ldots, a_m, 31$ are in A.P. The common difference, d is given by
$31 = 1 + (m+1)d \Rightarrow d = \frac{30}{m+1}$
Now, $\frac{a_7}{a_{m-1}} = \frac{5}{9} \Rightarrow \frac{1+7d}{1+(m-1)d} = \frac{5}{9}$
$\Rightarrow$ $\frac{5}{9} = \frac{1+\frac{210}{m+1}}{1+\frac{(m-1)30}{m+1}} = \frac{m+211}{31m-29}$
$\Rightarrow$ $5(31m - 29) = 9(m + 211) \Rightarrow m = 14$

11. (c) : Let the three numbers be $a - d, a, a + d\ (d > 0)$.
Now, $(a - d)\, a(a + d) = 224 \Rightarrow a(a^2 - d^2) = 224$ …(i)

Also, $a + d = 7\,(a - d) \Rightarrow d = \frac{3a}{4}$

Substituting this value of d in (i), we get

$$a\left(a^2 - \frac{9a^2}{16}\right) = 224 \Rightarrow a = 8 \text{ and } d = \frac{3}{4} \times 8 = 6$$

Hence, the three numbers are 2, 8 and 14.

12. (a) : Let a be the first term and d be the common difference.

$\therefore\ a_6 = 18 \Rightarrow a + 5d = 18$ ...(i)

$a_9 = 12 \Rightarrow a + 8d = 12$ ...(ii)

Solving (i) and (ii), we get

$a = 28, d = -2$

$\therefore\ a_{15} = a + 14d = 28 + 14\,(-2) = 0$

13. (a) : $T_n = S_n - S_{n-1}$

$\Rightarrow\ T_n = 5n^2 + 6n - 5(n - 1)^2 - 6(n - 1) = 10n + 1$

$\therefore\ T_r = 10r + 1 = 401 \Rightarrow 10r = 400 \Rightarrow r = 40$

14. (b) : $2 \log (2^x - 1) = \log 2 + \log(2^x + 3)$

$\Rightarrow\ (2^x - 1)^2 = 2 \cdot (2^x + 3) \Rightarrow (2^x)^2 - 4 \cdot 2^x - 5 = 0$

$\Rightarrow\ (2^x - 5)\,(2^x + 1) = 0 \Rightarrow x = \log_2 5\ (\because\ 2^x + 1 \neq 0)$

15. (b) : Let a be the first term and r be the common ratio. Then, $a_3 = 42 \Rightarrow ar^2 = 42$ ...(i)

$\therefore$ Product of first five terms $= a_1a_2a_3a_4a_5$

$= a(ar)(ar^2)(ar^3)(ar^4) = a^5r^{10} = (ar^2)^5 = (42)^5$[Using(i)]

16. (a) : The given progression is clearly a G.P. with first term $a = 1/4$ and common ratio $r = -2$

$\therefore$ 9th term $= a_9 = ar^8 = \frac{1}{4}(-2)^8 = 64$ and general term $= a_n = ar^{n-1} = \frac{1}{4}(-2)^{n-1} = (-1)^{n-1}\, 2^{n-3}$

17. (b) : Let $\frac{12}{r^2}, \frac{12}{r}, 12$ be a decreasing G.P.

$\therefore$ By data $\frac{12}{r^2}, \frac{12}{r}, 9$ are in A.P. $\Rightarrow \frac{24}{r} = 9 + \frac{12}{r^2}$

$\Rightarrow\ 3r^2 - 8r + 4 = 0 \Rightarrow r = 2 \text{ or } \frac{2}{3} \Rightarrow r = \frac{2}{3}$ only

($\because$ The given G.P. is a decreasing one)

18. (d) : Let the equation be

$x^2 - (\alpha + \beta)x + \alpha\beta = 0$ has roots α, β.

Also, $A = \frac{\alpha+\beta}{2}$, $G = \sqrt{\alpha\beta}$

$\Rightarrow\ \alpha + \beta = 2A, \alpha\beta = G^2$

$\therefore$ The required equation is $x^2 - 2Ax + G^2 = 0$.

19. (b) : Let the two numbers be a and b.

$x = \text{A.M.} = \frac{a+b}{2}$

$a, y = ar, z = ar^2, b = ar^3$ are in G.P.

$$\text{Now, } \frac{y^3 + z^3}{xyz} = \frac{a^3r^3 + a^3r^6}{\frac{(a+b)}{2} \cdot a^2r^3} = \frac{2a(1+r^3)}{a + ar^3} = 2$$

20. (b) : A.M. ≥ G.M. $\Rightarrow \frac{1 + x^{2m}}{2} \geq x^m$,

and $\frac{1 + y^{2n}}{2} \geq y^n$

$$\therefore\ \frac{x^m}{1 + x^{2m}} \cdot \frac{y^n}{1 + y^{2n}} \leq \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{4}$$

21. (a) : For $n = 441$, which is the square of natural number 21, we have $t_n = 2.7$

$\therefore\ t_{441} = 2.7$

For $n = 440$, which is not the square of a natural number, we have

$$t_n = \frac{n}{\frac{n}{44} - 1},\ \therefore\ t_{440} = \frac{440}{\frac{440}{44} - 1} = \frac{440}{9}$$

22. (b) : Given, a, b, c, d and e are in A.P.

Let D be the common difference of A.P.

Then, $b - a = c - b = d - c = e - d = D$ ...(i)

Now, consider $a - 4b + 6c - 4d + e$

$= a - b - 3b + 3c + 3c - 3d - d + e$

$= -(b - a) + 3\,(c - b) - 3\,(d - c) + (e - d)$

$= -D + 3D - 3D + D$ [Using (i)]

$= 0$

23. (d) : As $-\frac{2}{7}, k, -\frac{7}{2}$ are in G.P.

$$\therefore\ k^2 = -\left(\frac{2}{7}\right) \times -\left(\frac{7}{2}\right) = 1 \Rightarrow k = \pm 1$$

24. (a) : After each year, the value of the machine is 80% of its value in the previous year, so at the end of 5 years, the machine will depreciate as many times as 5. Hence, we have to find the 6th term of the G.P. whose first term a is 1250 and common ratio r is 0.8.

Hence, value at the end of 5 years $= t_6 = ar^5 = 1250(0.8)^5$
= ₹ 409.60

25. (a) : Here $a = 5,\ r = \frac{1}{3}$

$$\therefore\ S_\infty = \frac{a}{1 - r} = \frac{5}{1 - \frac{1}{3}} = \frac{15}{2} = 7.5$$

26. (b) : $g = \sqrt{ab}$, a, p, q, b are in A.P.

Common difference $(d) = \frac{b-a}{3}$

$\therefore \quad p = a + d = a + \frac{b-a}{3} = \frac{2a+b}{3}$

$q = b - d = b - \frac{b-a}{3} = \frac{a+2b}{3}$

So, $(2p-q)(p-2q) = \frac{(4a+2b-a-2b)}{3} \cdot \frac{(2a+b-2a-4b)}{3}$

$= -ab = -g^2$

$\therefore \quad \frac{(2p-q)(p-2q)}{g^2} = \frac{-g^2}{g^2} = -1$

27. (b) : Given $a + c = 2b$

Also, $\frac{a+b+c}{3} \geq \sqrt[3]{abc} = \sqrt[3]{64} = 4$

$\Rightarrow \quad \frac{3b}{3} \geq 4 \Rightarrow b \geq 4 \Rightarrow$ Minimum value of $b = 4$

28. (c) : We are given that, $\frac{a+2a}{2} = \sqrt{2a^2} + 2$

$\Rightarrow \quad \frac{3a}{2} = \sqrt{2}a + 2 \Rightarrow (3a-4)^2 = (2\sqrt{2}a)^2$

$\Rightarrow \quad a^2 - 24a + 16 = 0 \Rightarrow a = 4\left(3 \pm 2\sqrt{2}\right)$

29. (b) : Let S_n denote the sum of n terms and a_n be the n^{th} term of the given A.P.

Then, $S_n = 3n^2 + 5n$

$\Rightarrow \quad S_{n-1} = 3(n-1)^2 + 5(n-1) = 3n^2 - n - 2$

Now, $a_n = S_n - S_{n-1}$

$\Rightarrow \quad a_n = (3n^2 + 5n) - (3n^2 - n - 2) \Rightarrow a_n = 6n + 2$

Now, $a_m = 164 \Rightarrow 6m + 2 = 164 \Rightarrow 6m = 162$

$\Rightarrow \quad m = 27$

30. (c) : Given expression can be written as

$2\log_2 a + 4\log_2 a + 6\log_2 a + \ldots + 40\log_2 a$

$= \log_2 a\,\{2 + 4 + \ldots + 40\} = 10(2 + 40)\log_2 a = 420 \log_2 a$

Now, $420 \log_2 a = 840 \Rightarrow \log_2 a = 2 \Rightarrow a = 4$

31. (c) : Number of terms in set 1 $(S_1) = 1$
Number of terms in set 2 $(S_2) = 3$
Number of terms in set 3 $(S_3) = 5$
Number of terms upto $S_{20} = 1 + 3 + 5 + \ldots\ldots$ to 20 terms

$= \frac{20}{2}[2 + (19)2] = 400 \quad \therefore \; S_{21}$ starts with 401.

32. (a) : Let A_1, A_2, A_3 be 3 A.M.'s between 3 and 19. Then $3, A_1, A_2, A_3, 19$ are in A.P. whose common difference is

$d = \frac{19-3}{4} = 4$

$\therefore \quad A_1 = 3 + d \Rightarrow A_1 = 7,\; A_2 = 3 + 2d \Rightarrow A_2 = 11$
$A_3 = 3 + 3d \Rightarrow A_3 = 15$

Hence, the required A.M.'s are 7, 11 and 15.

33. (c) : Since, the sum is 28, we take the A.P. as $7 - 3d$, $7 - d$, $7 + d$, $7 + 3d$.

It is given that

$\frac{(7-3d)(7+d)}{(7-d)(7+3d)} = \frac{8}{15}$

$\Rightarrow \quad 15[49 - 14d - 3d^2] = 8[49 + 14d - 3d^2]$

$\Rightarrow \quad 343 - 322d - 21d^2 = 0$

$\Rightarrow \quad 3d^2 + 46d - 49 = 0 \Rightarrow d = 1$

$\therefore$ The four parts are 4, 6, 8, 10.

Hence, the largest part is 10.

34. (d) : $t_n = t_{n+1} + t_{n+2}$

$\Rightarrow \quad ar^{n-1} = ar^n + ar^{n+1} \Rightarrow 1 = r + r^2$

$\Rightarrow \quad r = \frac{-1 \pm \sqrt{5}}{2}$, since $r > 0$

Therefore, $r = 2\left(\frac{\sqrt{5}-1}{4}\right) = 2\sin 18°$

35. (d) : Given, $a_1 > 0$ and a_1, a_2, a_3 are in G.P. such that $a_2 = a_1 r$ and $a_3 = a_1 r^2$

$\therefore \quad 9a_1 + 5a_3 > 14a_2 \Rightarrow 9a_1 + 5a_1r^2 > 14a_1 r$

Since $a_1 > 0$, we get

$9 + 5r^2 > 14r \Rightarrow 5r^2 - 14r + 9 > 0$

$\Rightarrow \quad (5r - 9)(r - 1) > 0$

$\Rightarrow \quad r \in (-\infty, 1) \cup \left(\frac{9}{5}, \infty\right) \Rightarrow r \notin \left[1, \frac{9}{5}\right]$

36. (d) : Since, $b^2 = ca$, $c^2 = bd$, $bc = ad$

$\therefore \quad (b - c)^2 + (c - a)^2 + (b - d)^2$

$= b^2 - 2bc + c^2 + c^2 - 2ca + a^2 + b^2 - 2bd + d^2$

$= a^2 + d^2 - 2bc = a^2 + d^2 - 2ad = (a - d)^2$

37. (a) : $b = ar$, $c = ar^2$, $d = ar^3$

$\therefore \quad a^2 + b^2 + c^2 = a^2(1 + r^2 + r^4)$

$b^2 + c^2 + d^2 = a^2r^2(1 + r^2 + r^4)$

$ab + bc + cd = a^2 r(1 + r^2 + r^4)$

$\therefore \quad \frac{(a^2+b^2+c^2)(b^2+c^2+d^2)}{(ab+bc+cd)^2} = 1$

38. (b) : Here, $a = 2$, $r = 2$ and $n = 10$

$\therefore \quad S_n = \frac{a(r^n - 1)}{r - 1} \Rightarrow S_{10} = 2(2^{10} - 1) = 2046$

Hence, the number of ancestors preceding the person is 2046.

39. (c) : The $(n + 1)^{\text{th}}$ G.M. = middle G.M. = G.M. of 4 and 2916 $= \sqrt{4 \times 2916} = 108$

40. (c) : Given that A.M. = $\frac{a+b}{2} = 10$

$\Rightarrow \quad a + b = 20$ ...(i)

and G.M. = $\sqrt{ab} = 8 \Rightarrow ab = 64$

Now, $(a - b)^2 = (a + b)^2 - 4ab = 400 - 256 = 144$

$\Rightarrow \quad a - b = \pm 12$ ...(ii)

Solving (i) and (ii), we obtain

$a = 4, b = 16$ or $a = 16, b = 4$

Thus, the numbers a and b are 4, 16 or 16, 4.

41. (c) : $\frac{a+b}{2} + \sqrt{ab} = a - b \Rightarrow 2\sqrt{ab} = a - 3b$

$\Rightarrow \quad 4ab = (a - 3b)^2 \Rightarrow a^2 - 10ab + 9b^2 = 0$

$\Rightarrow \quad (a - 9b)(a - b) = 0 \Rightarrow a = 9b \quad [\because a \neq b]$

$\Rightarrow \frac{a}{b} = \frac{9}{1}$

42. (a) : Let a, g_1, g_2, b are in G.P.

$\therefore \quad g_1^2 = ag_2 \Rightarrow \frac{g_1^2}{g_2} = a$ ...(i)

Also, $g_2^2 = bg_1 \Rightarrow \frac{g_2^2}{g_1} = b$ ...(ii)

Adding (i) and (ii), we get

$\frac{g_1^2}{g_2} + \frac{g_2^2}{g_1} = a + b = 2a_1 \quad [\because a, a_1, b$ are in A.P.$]$

43. (a) : This is not a G.P., however, we can relate it to a G.P. by writing the terms as

$S_n = 7 + 77 + 777 + 7777 + \ldots$ to n terms

$= \frac{7}{9}\,[9 + 99 + 999 + 9999 + \ldots$ to n terms$]$

$= \frac{7}{9}[(10 - 1) + (10^2 - 1) + (10^3 - 1) + (10^4 - 1) + \ldots n$ terms$]$

$= \frac{7}{9}\,[(10 + 10^2 + 10^3 + \ldots n$ terms$) - (1 + 1 + 1 + \ldots n$ terms$)]$

$$= \frac{7}{9}\left[\frac{10(10^n - 1)}{10 - 1} - n\right] = \frac{7}{9}\left[\frac{10(10^n - 1)}{9} - n\right]$$

44. (b) : Let the sum of n terms of the given series is 5461.

Here, $a = 1$, $r = 4$ and $S_n = 5461$

So, $a\left(\frac{r^n - 1}{r - 1}\right) = 5461 \Rightarrow \frac{4^n - 1}{4 - 1} = 5461$

$\Rightarrow \quad 4^n - 1 = 16383 \Rightarrow 4^n = 16384 \Rightarrow 4^n = 4^7 \Rightarrow n = 7$

45. (c) : Let us consider a G.P. $a, ar, ar^2, \ldots$ with $2n$ terms,

we have, $\frac{a(r^{2n} - 1)}{r - 1} = \frac{5a\left((r^2)^n - 1\right)}{r^2 - 1}$

(since, common ratio of odd terms will be r^2 and number of terms will be n)

$\Rightarrow \quad \frac{a(r^{2n} - 1)}{r - 1} = 5\frac{a(r^{2n} - 1)}{(r^2 - 1)}$

$\Rightarrow a(r + 1) = 5a \Rightarrow r = 4$

46. (b) : We have, $a_2 = 1200$

$\Rightarrow a + d = 1200$ ...(i)

and $a_6 = 1400$

$\Rightarrow \; a + 5d = 1400$ ...(ii)

Subtracting (i) and (ii), we get

$4d = 200 \;\Rightarrow\; d = 50$

47. (d) : From (i), $a + 50 = 1200 \Rightarrow a = 1150$ units

48. (a) : Number of L.E.D. TV units manufactured in 10th year $= a_{10} = a + 9d = 1150 + 9 \times 50 = 1600$ units

49. (b) : Total production in 10 years = S_{10}

$= \frac{10}{2}[2 \times 1150 + (9)(50)] \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$= 5[2300 + 450]$

$= 5 \times 2750 = 13750$ units

50. (a) : $S_n = qn^2$

$\therefore \quad S_1 = q, S_2 = 4q, S_3 = 9q$ and $S_4 = 16q$

Now, $T_1 = q$

$\therefore \quad T_2 = S_2 - S_1 = 4q - q = 3q$

$T_3 = S_3 - S_2 = 9q - 4q = 5q$

$T_4 = S_4 - S_3 = 16q - 9q = 7q$

So, $T_m = (2m - 1)q$, m is any natural number.

$\Rightarrow \quad T_q = (2q - 1)q$

◆ ◆

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MONTHLY TEST DRIVE

XI

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Series 4 : Permutations and Combinations, Mathematical Reasoning, Principle of Mathematical Induction

Total Marks : 80 **Time Taken : 60 Min.**

Only One Option Correct Type

1. The greatest positive integer, which divides $n(n+1)(n+2)\ ...\ (n+r-1)\ \forall\ n \in N$ is
   (a) $r!$ (b) $(r+1)!$
   (c) $n+r$ (d) $n-r+1$

2. Number of diagonals in a decagon is
   (a) 45 (b) 36
   (c) 46 (d) 35

3. ABC is a triangle such that 4, 5, 6 points are marked on the sides AB, BC, CA respectively, the number of triangles having vertices on different sides is
   (a) $(4+5+6)!$ (b) $(4-1)(5-1)(6-1)$
   (c) $5!\ 4!\ 6!$ (d) $4 \times 5 \times 6$

4. Consider the following compound statements.
   (i) Mumbai is the capital of Rajasthan or Maharashtra.
   (ii) $\sqrt{3}$ is a rational number or an irrational number.
   (iii) 125 is a multiple of 7 or 8.
   (iv) A rectangle is a quadrilateral or a regular hexagon.
   Which of the above statements is not true ?
   (a) (i) (b) (ii) (c) (iii) (d) (iv)

5. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to
   (a) $s \vee (r \vee \sim s)$ (b) $s \wedge r$
   (c) $s \wedge \sim r$ (d) $s \wedge (r \wedge \sim s)$

6. The statement $P(r) = \sum_{r=1}^{n} r(r!) = (n+1)! - 1!$ is true for
   (a) $r > 2$ (b) $\forall\ r \in N$
   (c) for no value of r (d) $r > 1$

One or More Than One Option(s) Correct Type

7. There are 15 bulbs in a room. Each one of them can operated independently. The number of ways in which the room can be lighted, is
   (a) $8^5 - 1$ (b) $(32)^2 - 1$
   (c) $(32)^3 - 1$ (d) $8^4 - 1$

8. If $\binom{n}{r+1} = 56,\ \binom{n}{r} = 28,\ \binom{n}{r-1} = 8$, then
   (a) $r = 3$ (b) $r = 2$ (c) $n = 8$ (d) $n = 9$

9. The number of positive solutions of $x + y + z + u + v \leq 15$ and $x + y + z = 7$, is divisible by
   (a) 2 (b) 3 (c) 5 (d) 7

10. If $x = 100$, then
    (a) $E_2(x!) = 3$ less than x
    (b) The number of divisor of $E_5(x!) = 8$
    (c) $E_{10}(x!) = 24$
    (d) $E_{10}(x!)$ is minimum of $E_2(x!)$ and $E_5(x!)$

11. Let $P(n) : n^3 + n = 3m$, where m is a positive integer, then which of the following does not holds true?
    (a) $P(1)$ (b) $P(2)$ (c) $P(3)$ (d) $P(4)$

12. Let $P(n) = 2^{3n} - 7n - 1$, then $P(n)$ is not divisible by
    (a) 63 (b) 36 (c) 49 (d) 25

13. The following statement $(p \rightarrow q) \rightarrow [(\sim p \rightarrow q) \rightarrow q]$ is
    (a) equivalent to $\sim p \rightarrow q$
    (b) equivalent to $p \rightarrow q$
    (c) a fallacy (d) a tautology

Comprehension Type

Paragraph for Q. No. 14 and 15

Let S be the set of the first 18 natural numbers. The number of ways of selecting 3 numbers from S such that

18+

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14. they are all consecutive or none of them are consecutive is

(a) 576 (b) 600

(c) 640 (d) 680

15. they form an A.P. is

(a) 60 (b) 64

(c) 72 (d) 80

Matrix Match Type

16. Match the following :

	Column-I		Column-II
P.	The number of ways of choosing at least 3 students from a class of 12 students is	1.	256
Q.	The number of solutions of $x_1\, x_2\, x_3\, x_4 = 210$ is	2.	126
R.	The number of five digit numbers in which every digit exceeds the immediately preceeding digit is	3.	4017

	P	Q	R
(a)	3	1	2
(b)	1	2	4
(c)	4	3	1
(d)	2	4	3

Numerical Answer Type

17. The number of divisors of the form $4n + 2 (n \geq 0)$ of the integer 240 is ______.

18. A child attempts to open a five disc-lock. He takes 5 sec time to dial a particular number on the disc. If he does so for 5 hrs. every day, then the number of days, he would be sure to open the lock, is ______.

19. For any positive integer n, $n^3 + 2n$ is always divisible by ______.

20. If a, b are any two odd positive integers such that $a > b$, then the largest positive integer which divides all the numbers of the form $a^2 - b^2$ is ______.

⁂

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SELF CHECK

No. of questions attempted ……

No. of questions correct ……

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90-75%	**GOOD WORK !**	You can score good in the final exam.
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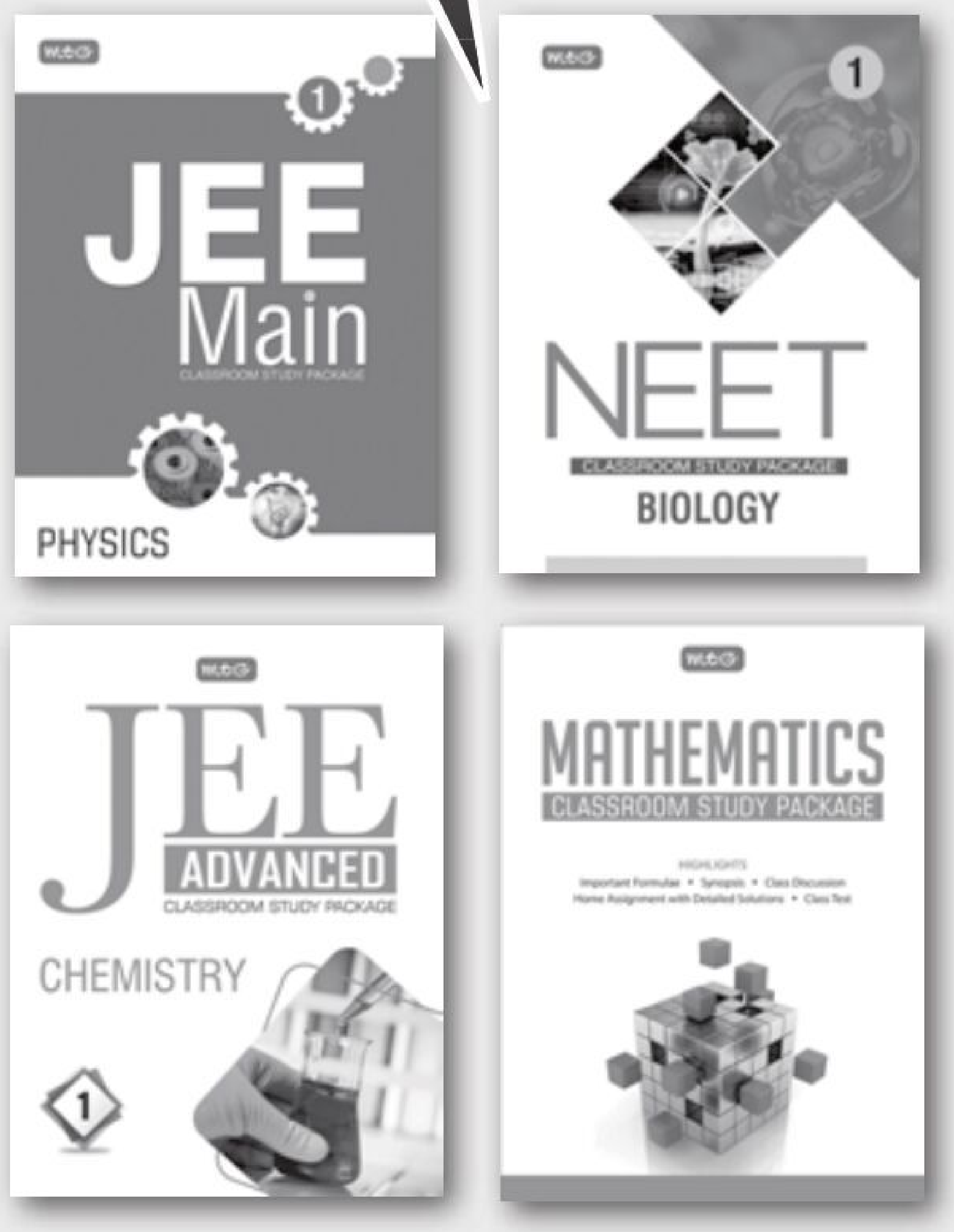

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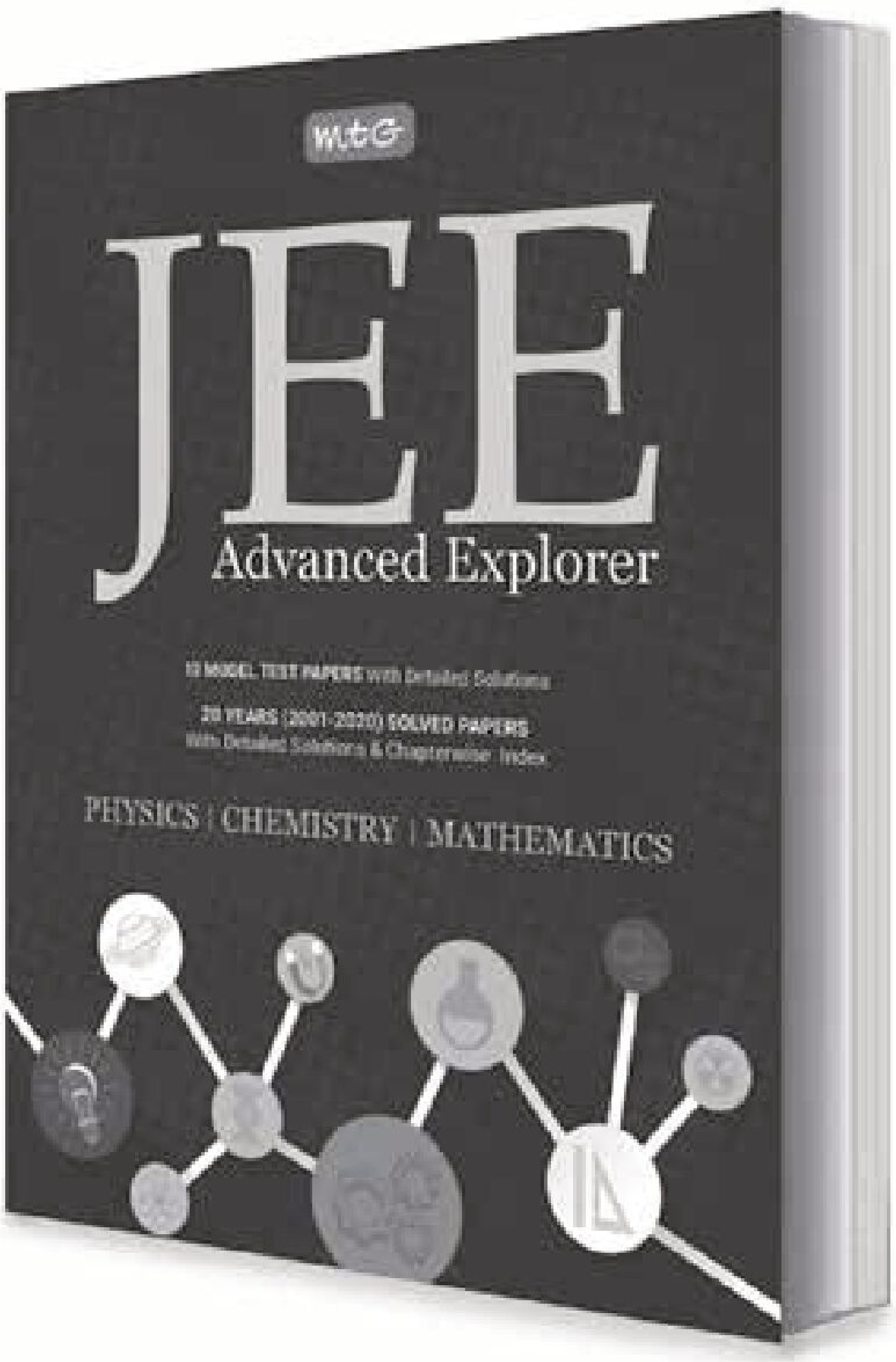
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CBSE warm-up!

CLASS-XII

TERM-I OBJECTIVE TYPE QUESTIONS*

Series 3 Matrices

Time Allowed : 90 min.
Maximum Marks : 40

General Instructions :

1. *This question paper contains three sections A, B and C. Each part is compulsory.*
2. *Section - A has 20 MCQs, attempt any 16 out of 20.*
3. *Section - B has 20 MCQs, attempt any 16 out of 20.*
4. *Section - C has 10 MCQs, attempt any 8 out of 10.*
5. *There is no negative marking.*
6. *All questions carry equal marks.*

SECTION - A

In this section, attempt any 16 questions out of Questions 1-20. Each Question is of 1 mark weightage.

1. A square matrix $A = [a_{ij}]_{n \times n}$ is called a diagonal matrix if $a_{ij} = 0$ for

(a) $i = j$ (b) $i < j$
(c) $i > j$ (d) $i \neq j$

2. If $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$, then the values of x, y, z and w respectively are

(a) 2, 2, 3, 4 (b) 2, 3, 1, 2
(c) 3, 3, 0, 1 (d) None of these

3. If $A = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}$ and $A^2 - kA - 5I = O$, then $k =$

(a) 5 (b) 3
(c) 7 (d) None of these

4. If $AB = A$ and $BA = B$, then

(a) $B = I$ (b) $A = I$
(c) $A^2 = A$ (d) $B^2 = I$

5. If A is a $m \times n$ matrix such that AB and BA are both defined, then B is a

(a) $m \times n$ matrix (b) $n \times m$ matrix
(c) $n \times n$ matrix (d) $m \times m$ matrix

6. If $A = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$, then A^2 is

(a) null matrix (b) unit matrix
(c) $\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$

7. If $A = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$,

then AB is equal to

***Chapterwise practice questions for CBSE Exam Term- I as per the pattern issued by CBSE.**

(a) $\begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$ (b) $\begin{bmatrix} 11 & 4 & 3 \\ 1 & 2 & 3 \\ 0 & 3 & 3 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & 8 & 4 \\ 2 & 9 & 6 \\ 0 & 2 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 & 2 \\ 5 & 4 & 3 \\ 1 & 8 & 2 \end{bmatrix}$

8. If A is a square matrix such that $A^2 = A$, then $(I + A)^3 - 7A$ is equal to

(a) A (b) $I - A$ (c) I (d) $3A$

9. If $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix}$ and $A = A^T$, then

(a) $x = 0, y = 5$ (b) $x + y = 5$
(c) $x = y$ (d) $x = 0, y = 9$

10. If $A = \begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix}$, then $A + 2A^T$ equals

(a) A (b) $-A^T$ (c) A^T (d) $2A^2$

11. If A and B are symmetric matrices of the same order, then

(a) AB is a symmetric matrix
(b) $A - B$ is a skew-symmetric matrix
(c) $AB + BA$ is a symmetric matrix
(d) $AB - BA$ is a symmetric matrix

12. If $A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$ is expressed as the sum of a symmetric matrix B and skew-symmetric matrix C, then C is

(a) $\begin{bmatrix} 2 & 2 & -4 \\ 2 & 3 & 4 \\ -4 & 4 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 4 & -5 \\ 0 & 3 & 7 \\ -3 & 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 0 & -2 & 1 \\ 2 & 0 & -3 \\ -1 & 3 & 0 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

13. If the matrix $A = \begin{bmatrix} 5 & 2 & x \\ y & 2 & -3 \\ 4 & t & -7 \end{bmatrix}$ is a symmetric matrix, then find the values of x, y and t respectively.

(a) 4, 2, 3 (b) 4, 2, –3
(c) 4, 2, –7 (d) 2, 4, –7

14. If $A = \begin{bmatrix} 2x & 0 \\ x & x \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$, then x equals to

(a) 2 (b) $-\frac{1}{2}$ (c) 1 (d) $\frac{1}{2}$

15. If $A^3 = O$, then $A^2 + A + I =$

(a) $I - A$ (b) $(I - A)^{-1}$
(c) $(I + A)^{-1}$ (d) $I + A$

16. If $A^2 - A + I = O$, then the inverse of A is

(a) $I - A$ (b) $A - I$
(c) A (d) $A + I$

17. If a matrix A is both symmetric and skew-symmetric, then

(a) A is a diagonal matrix
(b) A is a zero matrix
(c) A is a scalar matrix
(d) A is a square matrix

18. If A is a symmetric matrix and $n \in N$, then A^n is

(a) symmetric matrix
(b) a diagonal matrix
(c) skew-symmetric matrix
(d) None of these

19. If $A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$ and $f(x) = (1 + x)(1 - x)$, then $f(A)$ is

(a) $-4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (b) $-8\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

(c) $4\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ (d) None of these

20. If $A = \begin{bmatrix} a & b \\ b & a \end{bmatrix}$ and $A^2 = \begin{bmatrix} \alpha & \beta \\ \beta & \alpha \end{bmatrix}$, then

(a) $\alpha = a^2 + b^2, \beta = ab$
(b) $\alpha = a^2 + b^2, \beta = 2ab$
(c) $\alpha = a^2 + b^2, \beta = a^2 - b^2$
(d) $\alpha = 2ab, \beta = a^2 + b^2$

SECTION - B

In this section, attempt any 16 questions out of the Questions 21 - 40. Each Question is of 1 mark weightage.

21. If $\begin{bmatrix} a+b & 2 \\ 5 & ab \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$, then find the values of a and b respectively.

(a) 2, 4 (b) 4, 2
(c) Both (a) and (b) (d) None of these

22. $\begin{bmatrix} 7 & 1 & 2 \\ 9 & 2 & 1 \end{bmatrix}\begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}+2\begin{bmatrix} 4 \\ 2 \end{bmatrix}$ is equal to

(a) $\begin{bmatrix} 43 \\ 44 \end{bmatrix}$ (b) $\begin{bmatrix} 43 \\ 45 \end{bmatrix}$ (c) $\begin{bmatrix} 45 \\ 44 \end{bmatrix}$ (d) $\begin{bmatrix} 44 \\ 45 \end{bmatrix}$

23. Construct a 2 × 2 matrix $A = [a_{ij}]$ whose elements a_{ij} are given by $a_{ij} = \dfrac{|-3i+j|}{2}$.

(a) $\begin{bmatrix} 1 & 1/2 \\ 5/2 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & -1/2 \\ -5/2 & -2 \end{bmatrix}$

(c) $\begin{bmatrix} 1 & -1/2 \\ -5/2 & 2 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & 1/2 \\ -5/2 & -2 \end{bmatrix}$

24. Solve the matrix equation $[x \quad 1]\begin{bmatrix} 1 & 0 \\ -2 & -3 \end{bmatrix}\begin{bmatrix} x \\ 5 \end{bmatrix}=O.$

(a) 3, 5 (b) −3, 5 (c) 3, −5 (d) −3, −5

25. If $B=\begin{bmatrix} -1 & 0 & 2 \\ 3 & 4 & 1 \end{bmatrix}, C=\begin{bmatrix} -1 & 2 & 3 \\ 2 & 1 & 0 \end{bmatrix},$

then find $B + C$.

(a) $\begin{bmatrix} -2 & -2 & -5 \\ -5 & -5 & -1 \end{bmatrix}$ (b) $\begin{bmatrix} 2 & 2 & 5 \\ 5 & 5 & -1 \end{bmatrix}$

(c) $\begin{bmatrix} -2 & 2 & 5 \\ 5 & 5 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} -2 & -2 & -5 \\ 5 & 5 & 1 \end{bmatrix}$

26. For what values of x and y, the matrices

$A=\begin{bmatrix} 2x+1 & 3y \\ 0 & y^2-5y \end{bmatrix}, B=\begin{bmatrix} x+3 & y^2+2 \\ 0 & -6 \end{bmatrix}$

will be equal?

(a) 2, 3 (b) 3, 4 (c) 2, 2 (d) 3, 3

27. The order of the single matrix obtained from

$\begin{bmatrix} 1 & -1 \\ 0 & 2 \\ 2 & 3 \end{bmatrix}\left\{\begin{bmatrix} -1 & 0 & 2 \\ 2 & 0 & 1 \end{bmatrix}-\begin{bmatrix} 0 & 1 & 23 \\ 1 & 0 & 21 \end{bmatrix}\right\}$ is

(a) 2 × 3 (b) 2 × 2 (c) 3 × 2 (d) 3 × 3

28. If $A=\begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix}$, then $(AB)^T$ is equal to

(a) $\begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$ (b) $\begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

(c) $\begin{bmatrix} -3 & 7 \\ 10 & 2 \end{bmatrix}$ (d) None of these

29. If $A^T=\begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ and $B=\begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$,

then find $A^T - B^T$.

(a) $\begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$ (b) $\begin{bmatrix} 4 & 3 \\ 3 & 0 \\ 1 & 2 \end{bmatrix}$

(c) $\begin{bmatrix} 4 & 0 \\ -1 & -3 \\ 3 & -2 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & -3 \\ 2 & 4 \\ 3 & 5 \end{bmatrix}$

30. If a matrix has 12 elements, then it has ________ possible orders.

(a) 3 (b) 6 (c) 4 (d) 8

31. If $A=\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, then for what value of α, A is an identity matrix?

(a) 0° (b) 90° (c) 45° (d) 30°

32. Simplify :

$\tan\theta\begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & -\sec\theta \end{bmatrix}+\sec\theta\begin{bmatrix} -\tan\theta & -\sec\theta \\ -\sec\theta & \tan\theta \end{bmatrix}$

(a) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

(c) $\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

33. If $\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\dfrac{1}{40}\begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$, then the value of $x+y+z$ is

(a) 3 (b) 0 (c) 2 (d) 1

34. If A and B are two matrices of the order $3 \times m$ and $3 \times n$, respectively and $m = n$, then the order of matrix $(5A - 2B)$ is

(a) $m \times 3$ (b) 3×3 (c) $m \times n$ (d) $3 \times n$

35. Total number of possible matrices of order 3 × 3 with each entry 2 or 0 is

(a) 9 (b) 27 (c) 81 (d) 512

36. If $A=\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$ satisfies $A^4 = kA^T$, then find the value of k.

(a) 1 (b) 4 (c) 8 (d) 10

37. Each diagonal element of a skew-symmetric matrix is
(a) zero (b) positive
(c) non-real (d) negative

38. If $\begin{pmatrix} 2x+y & 3y \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 6 & 4 \end{pmatrix}'$, then $x =$
(a) 1 (b) –1 (c) 2 (d) –2

39. If $A = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$, then find the matrix $A^2 - B$.
(a) $\begin{bmatrix} 1 & -8 \\ 13 & -6 \end{bmatrix}$ (b) $\begin{bmatrix} 1 & 8 \\ 13 & -6 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & -8 \\ -13 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} 1 & 8 \\ 13 & 6 \end{bmatrix}$

40. If $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix}$, then find the value of α for which $A^2 = B$.
(a) 1 (b) –1
(c) 4 (d) No real values

SECTION - C

- *In this section, attempt any 8 questions.*
- *Each question is of 1-mark weightage.*
- *Questions 46-50 are based on a Case-Study.*

41. If $A = [a_{ij}]_{2\times 2}$, where $a_{ij} = \dfrac{(i+2j)^2}{2}$, then A is equal to
(a) $\begin{bmatrix} 9 & 25 \\ 8 & 18 \end{bmatrix}$ (b) $\begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$
(c) $\begin{bmatrix} 9 & 25 \\ 4 & 9 \end{bmatrix}$ (d) $\begin{bmatrix} 9/2 & 15/2 \\ 4 & 9 \end{bmatrix}$

42. If $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$, then the value of w is
(a) 1 (b) 2 (c) 3 (d) 4

43. If a square matrix $A = [a_{ij}]$, $a_{ij} = i^2 - j^2$ is of order 2×2, then
(a) A is a skew-symmetric matrix
(b) A is a symmetric matrix
(c) A is neither symmetric nor skew-symmetric
(d) Both (a) and (b)

44. The additive inverse of $A + B$, where A and B are given as $A = \begin{bmatrix} 2 & 5 \\ 9 & 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 \\ 3 & -9 \end{bmatrix}$ is
(a) $\begin{bmatrix} -1 & -7 \\ 12 & 6 \end{bmatrix}$ (b) $\begin{bmatrix} -1 & 7 \\ -12 & -6 \end{bmatrix}$
(c) $\begin{bmatrix} 1 & -7 \\ -12 & 6 \end{bmatrix}$ (d) $\begin{bmatrix} -1 & -7 \\ -12 & 6 \end{bmatrix}$

45. If $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$, then the value of k is
(a) 17 (b) 12 (c) 10 (d) 5

CASE STUDY

Consider 2 families A and B. Suppose there are 4 men, 4 women and 4 children in family A and 2 men, 2 women and 2 children in family B. The recommend daily amount of calories is 2400 for a man, 1900 for a woman, 1800 for a child and 45 grams of proteins for a man, 55 grams for a woman and 33 grams for a child.

Based on the given information, answer the following questions.

46. The requirement of calories and proteins for each person, in matrix form can be represented as

(a) $\begin{array}{l} \\ \text{Man} \\ \text{Woman} \\ \text{Child} \end{array}\begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Proteins} \end{array} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$

(b) $\begin{array}{l} \\ \text{Man} \\ \text{Woman} \\ \text{Child} \end{array}\begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Proteins} \end{array} \\ \begin{bmatrix} 1900 & 55 \\ 2400 & 45 \\ 1800 & 33 \end{bmatrix} \end{array}$

(c) $$\begin{array}{cc} & \begin{array}{cc}\text{Calories} & \text{Proteins}\end{array} \\ \begin{array}{l}\text{Man}\\ \text{Woman}\\ \text{Child}\end{array} & \begin{bmatrix}1800 & 33\\ 1900 & 55\\ 2400 & 45\end{bmatrix}\end{array}$$

(d) $$\begin{array}{cc} & \begin{array}{cc}\text{Calories} & \text{Proteins}\end{array} \\ \begin{array}{l}\text{Man}\\ \text{Woman}\\ \text{Child}\end{array} & \begin{bmatrix}2400 & 33\\ 1900 & 55\\ 1800 & 45\end{bmatrix}\end{array}$$

47. Requirement of calories of family A is

(a) 24000 (b) 24400

(c) 15000 (d) 15800

48. Requirement of proteins for family B is

(a) 560 grams (b) 332 grams

(c) 266 grams (d) 300 grams

49. If A and B are two matrices such that $AB = B$ and $BA = A$, then $A^2 + B^2$ equals

(a) $2AB$ (b) $2BA$ (c) $A + B$ (d) AB

50. If $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{n \times p}$ and $C = (c_{ij})_{p \times q}$, then the product $(BC)A$ is possible only when

(a) $m = q$ (b) $n = q$

(c) $p = q$ (d) $m = p$

SOLUTIONS

1. (d) : $A = [a_{ij}]_{n \times n}$ is a diagonal matrix iff all non-diagonal entries are 0 *i.e.*, $a_{ij} = 0$ for $i \neq j$.

2. (a) : Since, $\begin{bmatrix} x+y & 2x+z \\ x-y & 2z+w \end{bmatrix} = \begin{bmatrix} 4 & 7 \\ 0 & 10 \end{bmatrix}$

$\Rightarrow\ x + y = 4,\ x - y = 0,\ 2x + z = 7$ and $2z + w = 10$

On solving these equations, we get

$x = 2,\ y = 2,\ z = 3$ and $w = 4$

3. (a) : Given, $A^2 - kA - 5I = O$

$\Rightarrow\ kA = A^2 - 5I$

$$\Rightarrow\ kA = \begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 10 & 15 \\ 15 & 25 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 5 & 15 \\ 15 & 20 \end{bmatrix} = 5\begin{bmatrix} 1 & 3 \\ 3 & 4 \end{bmatrix} = 5A$$

$\Rightarrow\ kA = 5A\ \ \therefore\ \ k = 5$

4. (c) : $A = AB = A(BA) = (AB)A = A \cdot A = A^2$

$B = BA = B(AB) = (BA)B = B \cdot B = B^2$

5. (b) : Since AB exists, therefore, number of columns in A = number of rows in B. So, B has n rows. Since BA exists, number of columns in B = number of rows in A. So, B has m columns.

$\therefore\ \ B$ is a $n \times m$ matrix.

6. (a) : Given that, $A = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}$

$$\therefore\ A^2 = \begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -2 & 4 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$\Rightarrow\ A^2$ is a null matrix.

7. (a) : We have, $AB = \begin{bmatrix} 1 & 2 & -1 \\ 3 & 0 & 2 \\ 4 & 5 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

$$= \begin{bmatrix} 1+4+0 & 0+2-1 & 0+0-3 \\ 3+0+0 & 0+0+2 & 0+0+6 \\ 4+10+0 & 0+5+0 & 0+0+0 \end{bmatrix} = \begin{bmatrix} 5 & 1 & -3 \\ 3 & 2 & 6 \\ 14 & 5 & 0 \end{bmatrix}$$

8. (c) : We have, $A^2 = A$ ...(i)

Now, $(I + A)^3 - 7A = I^3 + A^3 + 3A^2I + 3AI^2 - 7A$

$= I + A^2A + 3A^2I + 3AI - 7A$

$= I + AA + 3A + 3A - 7A$ {Using (i)}

$= I + A^2 - A = I + A - A$ {Using (i)}

$= I$

9. (c) : We have, $A = \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix}$

Now, $A = A^T \Rightarrow \begin{bmatrix} 5 & x \\ y & 0 \end{bmatrix} = \begin{bmatrix} 5 & y \\ x & 0 \end{bmatrix} \Rightarrow x = y$

10. (c) : $A^T = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & -3 \\ 2 & 3 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & -1 & 2 \\ 1 & 0 & 3 \\ -2 & -3 & 0 \end{bmatrix} = -A$

So, $A^T = -A \Rightarrow A = -A^T$

Hence, $A + 2A^T = -A^T + 2A^T = A^T$

11. (c) : $(AB + BA)^T = (AB)^T + (BA)^T$

$= B^TA^T + A^TB^T = BA + AB$ $(\because A^T = A$ and $B^T = B)$

$= AB + BA$

Hence, $AB + BA$ is a symmetric matrix.

12. (c) : $A = \begin{bmatrix} 2 & 0 & -3 \\ 4 & 3 & 1 \\ -5 & 7 & 2 \end{bmatrix}$

$$A = \left(\frac{A + A'}{2}\right) + \left(\frac{A - A'}{2}\right) = B + C$$

[where B and C are symmetric and skew-symmetric matrices respectively]

$$\therefore \quad C=\frac{A-A'}{2}=\frac{1}{2}\left\{\begin{bmatrix}2&0&-3\\4&3&1\\-5&7&2\end{bmatrix}-\begin{bmatrix}2&4&-5\\0&3&7\\-3&1&2\end{bmatrix}\right\}$$

$$=\frac{1}{2}\begin{bmatrix}0&-4&2\\4&0&-6\\-2&6&0\end{bmatrix}=\begin{bmatrix}0&-2&1\\2&0&-3\\-1&3&0\end{bmatrix}$$

13. (b) : Since, A is a symmetric matrix $\therefore$ $A = A^T$

$$\Rightarrow \begin{bmatrix}5&2&x\\y&2&-3\\4&t&-7\end{bmatrix}=\begin{bmatrix}5&y&4\\2&2&t\\x&-3&-7\end{bmatrix}$$

On comparing, we get

$y = 2, x = 4, t = -3$

14. (d) : $A=\begin{bmatrix}2x&0\\x&x\end{bmatrix}, A^{-1}=\begin{bmatrix}1&0\\-1&2\end{bmatrix}$

We know that, $AA^{-1} = I$

$$\Rightarrow \begin{bmatrix}2x&0\\x&x\end{bmatrix}\begin{bmatrix}1&0\\-1&2\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix} \Rightarrow \begin{bmatrix}2x&0\\0&2x\end{bmatrix}=\begin{bmatrix}1&0\\0&1\end{bmatrix}$$

On comparing, we get

$2x = 1 \ \Rightarrow \ x=\frac{1}{2}$

15. (b) : Since, $A^3 = O$

$\therefore \quad I - A^3 = I \Rightarrow (I - A)(I + A + A^2) = I$

$\therefore \quad I + A + A^2 = (I - A)^{-1}$

16. (a) : We know that if A is any square matrix, then $AA^{-1} = I$ and $A^{-1}I = A^{-1}$

Now, $A^2 - A + I = O$

$\Rightarrow \quad A^{-1}A^2 - A^{-1}A + A^{-1}I = A^{-1}O$

$\Rightarrow \quad (A^{-1}A)A - (A^{-1}A) + A^{-1} = O$

$\Rightarrow \quad IA - I + A^{-1} = O \ \Rightarrow \ A - I + A^{-1} = O$

$\Rightarrow \quad A^{-1} = I - A$

17. (b) : A is a symmetric matrix

$\therefore \quad A' = A$ …(i)

A is also a skew-symmetric matrix

$\therefore \quad A' = -A$ …(ii)

From (i) and (ii), we get

$A = -A \ \Rightarrow \ A = O$

Hence, A is a zero matrix.

18. (a) : Since, A is a symmetric matrix

$\Rightarrow \quad A' = A$

Now, $(A^n)' = (A')^n = A^n$

Therefore, A^n is a symmetric matrix.

19. (a) : Given, $f(x) = (1 + x)(1 - x) = 1 - x^2$

$\Rightarrow \quad f(A) = I - A^2$ (Putting $x = A$)

$$\Rightarrow \quad f(A)=\begin{bmatrix}1&0\\0&1\end{bmatrix}-\left\{\begin{bmatrix}1&2\\2&1\end{bmatrix}\begin{bmatrix}1&2\\2&1\end{bmatrix}\right\}$$

$$=\begin{bmatrix}1&0\\0&1\end{bmatrix}-\begin{bmatrix}5&4\\4&5\end{bmatrix}=\begin{bmatrix}-4&-4\\-4&-4\end{bmatrix}=-4\begin{bmatrix}1&1\\1&1\end{bmatrix}$$

20. (b) : Given, $A=\begin{bmatrix}a&b\\b&a\end{bmatrix}, A^2=\begin{bmatrix}\alpha&\beta\\\beta&\alpha\end{bmatrix}$

Now, $A^2=A\cdot A=\begin{bmatrix}a&b\\b&a\end{bmatrix}\begin{bmatrix}a&b\\b&a\end{bmatrix}$

$$\Rightarrow \begin{bmatrix}\alpha&\beta\\\beta&\alpha\end{bmatrix}=\begin{bmatrix}a^2+b^2&2ab\\2ab&a^2+b^2\end{bmatrix}$$

$\Rightarrow \quad \alpha = a^2 + b^2, \beta = 2ab$

21. (c) : Since, $\begin{bmatrix}a+b&2\\5&ab\end{bmatrix}=\begin{bmatrix}6&2\\5&8\end{bmatrix}$

$\Rightarrow \quad a + b = 6$ and $ab = 8 \ \Rightarrow \ a + 8/a = 6$

$\Rightarrow \quad a^2 - 6a + 8 = 0 \ \Rightarrow \ (a - 2)(a - 4) = 0 \ \Rightarrow \ a = 2, 4$

Hence, $a = 2, b = 4$ or $a = 4, b = 2$

22. (a) : $\begin{bmatrix}7&1&2\\9&2&1\end{bmatrix}\begin{bmatrix}3\\4\\5\end{bmatrix}+2\begin{bmatrix}4\\2\end{bmatrix}=\begin{bmatrix}21+4+10\\27+8+5\end{bmatrix}+\begin{bmatrix}8\\4\end{bmatrix}$

$$=\begin{bmatrix}35\\40\end{bmatrix}+\begin{bmatrix}8\\4\end{bmatrix}=\begin{bmatrix}43\\44\end{bmatrix}$$

23. (a) : Here, $a_{ij}=\frac{|-3i+j|}{2}$

$$\therefore \quad a_{11}=\frac{|-3+1|}{2}=1, a_{12}=\frac{|-3+2|}{2}=\frac{1}{2},$$

$$a_{21}=\frac{|-6+1|}{2}=\frac{5}{2}, a_{22}=\frac{|-6+2|}{2}=2$$

$\therefore \quad$ Required matrix $A=\begin{bmatrix}1&1/2\\5/2&2\end{bmatrix}$

24. (b) : $[x \ \ 1]\begin{bmatrix}1&0\\-2&-3\end{bmatrix}\begin{bmatrix}x\\5\end{bmatrix}=O$

$$\Rightarrow \quad [x-2 \ \ -3]\begin{bmatrix}x\\5\end{bmatrix}=O$$

$\Rightarrow \quad x^2 - 2x - 15 = 0 \Rightarrow (x + 3)(x - 5) = 0 \Rightarrow x = -3, 5$

25. (c) : $B+C=\begin{bmatrix}-1&0&2\\3&4&1\end{bmatrix}+\begin{bmatrix}-1&2&3\\2&1&0\end{bmatrix}=\begin{bmatrix}-2&2&5\\5&5&1\end{bmatrix}$

26. (c) : Since, the corresponding elements of two equal matrices are equal.

i.e., $A = B \Rightarrow 2x + 1 = x + 3 \Rightarrow x = 2$,
$3y = y^2 + 2 \Rightarrow y^2 - 3y + 2 = 0 \quad \Rightarrow y = 1, 2$
and $y^2 - 5y = -6 \Rightarrow y^2 - 5y + 6 = 0 \Rightarrow y = 2, 3$
Since, $3y = y^2 + 2$ and $y^2 - 5y = -6$ must hold good simultaneously when we take the common solution of these two equations. Therefore $y = 2$.
Hence, $A = B$ if $x = 2$, $y = 2$

27. (d) : When a 3×2 matrix is post multiplied by a 2×3 matrix, the product is a 3×3 matrix.

28. (b) : $AB = \begin{bmatrix} 1 & -2 & 1 \\ 2 & 1 & 3 \end{bmatrix}\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} -3 & -2 \\ 10 & 7 \end{bmatrix}$

$\Rightarrow \quad (AB)^T = \begin{bmatrix} -3 & 10 \\ -2 & 7 \end{bmatrix}$

29. (a) : $B^T = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

So, $A^T - B^T = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix}$

30. (b) : All possible orders of 12 elements are
1 × 12, 12 × 1, 2 × 6, 6 × 2, 3 × 4, 4 × 3 *i.e.*, 6 in number.

31. (a) : We have, $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$,

A is an identity matrix if
$\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad \cos\alpha = 1$ and $\sin\alpha = 0 \Rightarrow \alpha = 0^\circ$

32. (b) : Given,

$$\tan\theta\begin{bmatrix} \sec\theta & \tan\theta \\ \tan\theta & -\sec\theta \end{bmatrix} + \sec\theta\begin{bmatrix} -\tan\theta & -\sec\theta \\ -\sec\theta & \tan\theta \end{bmatrix}$$

$$= \begin{bmatrix} \tan\theta\sec\theta & \tan^2\theta \\ \tan^2\theta & -\tan\theta\sec\theta \end{bmatrix} + \begin{bmatrix} -\tan\theta\sec\theta & -\sec^2\theta \\ -\sec^2\theta & \tan\theta\sec\theta \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}. \qquad [\because \sec^2\theta - \tan^2\theta = 1]$$

33. (a) : Given, $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 10 & -5 \\ -5 & -2 & 13 \\ 10 & -4 & 6 \end{bmatrix}\begin{bmatrix} 5 \\ 0 \\ 5 \end{bmatrix}$

$$= \frac{1}{40}\begin{bmatrix} 25+0-25 \\ -25+0+65 \\ 50+0+30 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 0 \\ 40 \\ 80 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$$

$\Rightarrow \quad x = 0, y = 1$ and $z = 2$
$\therefore \quad x + y + z = 0 + 1 + 2 = 3$

34. (d) : A is of order $3 \times m$ and B is of order $3 \times n$ and $m = n$
So, $5A - 2B$ is of order $3 \times m$ or $3 \times n$

35. (d) : Required number of matrices $= 2^9 = 512$

36. (c) : We have, $A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} \Rightarrow A^T = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$

$\therefore \quad A = A^T$

Now, $A^2 = A \cdot A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1+1 & 1+1 \\ 1+1 & 1+1 \end{bmatrix}$

$= \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix} = 2\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix} = 2A$

So, $A^4 = A^2 \cdot A^2 = 4A \cdot A = 4A^2 = 4 \cdot 2A = 8A = 8A^T$
$[\because A = A^T]$

Hence, $k = 8$

37. (a)

38. (c) : We have, $\begin{pmatrix} 2x+y & 3y \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 0 \\ 6 & 4 \end{pmatrix}'$

$\Rightarrow \begin{pmatrix} 2x+y & 3y \\ 0 & 4 \end{pmatrix} = \begin{pmatrix} 6 & 6 \\ 0 & 4 \end{pmatrix}$

By equality of two matrices, we have
$2x + y = 6$ and $3y = 6 \Rightarrow y = 2$.
Putting the value of y, we get
$2x + 2 = 6 \Rightarrow 2x = 4 \Rightarrow x = 2$

39. (a): $A^2 = \begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 4-3 & -2-2 \\ 6+6 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix}$

$\therefore \quad A^2 - B = \begin{bmatrix} 1 & -4 \\ 12 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 4 \\ -1 & 7 \end{bmatrix}$

$= \begin{bmatrix} 1-0 & -4-4 \\ 12+1 & 1-7 \end{bmatrix} = \begin{bmatrix} 1 & -8 \\ 13 & -6 \end{bmatrix}$

40. (d) : Given that, $A = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} \alpha & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2+0 & 0+0 \\ \alpha+1 & 0+1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$

Also, $B = A^2 \Rightarrow \begin{bmatrix} 1 & 0 \\ 5 & 1 \end{bmatrix} = \begin{bmatrix} \alpha^2 & 0 \\ \alpha+1 & 1 \end{bmatrix}$

Clearly, this is not satisfied by any real value of α.

41. (b) : Here,

$a_{11} = \frac{(1+2\times1)^2}{2} = \frac{9}{2}$, $a_{12} = \frac{(1+2\times2)^2}{2} = \frac{25}{2}$,

$a_{21} = \frac{(2+2\times1)^2}{2} = 8$ and $a_{22} = \frac{(2+2\times2)^2}{2} = 18$

So, the required matrix $A = \begin{bmatrix} 9/2 & 25/2 \\ 8 & 18 \end{bmatrix}$.

42. (d) : Since, $\begin{bmatrix} x-y & 2x+z \\ 2x-y & 3z+w \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$

$\Rightarrow \quad x - y = -1,\ 2x - y = 0,\ 2x + z = 5$, and $3z + w = 13$

On solving these equations, we get

$x = 1,\ y = 2,\ z = 3$ and $w = 4$

43. (a) : We have, $a_{11} = 1^2 - 1^2 = 0$

$a_{12} = 1^2 - 2^2 = -3,\ a_{21} = 2^2 - 1^2 = 3,\ a_{22} = 2^2 - 2^2 = 0$

Now, $A = \begin{bmatrix} 0 & -3 \\ 3 & 0 \end{bmatrix}$ and $A^T = \begin{bmatrix} 0 & 3 \\ -3 & 0 \end{bmatrix}$

$\therefore \quad A^T = -A$ therefore A is a skew symmetric matrix.

44. (d) : Let $C = A + B = \begin{bmatrix} 1 & 7 \\ 12 & -6 \end{bmatrix}$

Now, $(-C) = \begin{bmatrix} -1 & -7 \\ -12 & 6 \end{bmatrix}$ is the additive inverse of $A + B$.

45. (a) : Given, $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} 1\cdot3+2\cdot2 & 1\cdot1+2\cdot5 \\ 3\cdot3+4\cdot2 & 3\cdot1+4\cdot5 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 7 & 11 \\ 17 & 23 \end{bmatrix} = \begin{bmatrix} 7 & 11 \\ k & 23 \end{bmatrix} \Rightarrow k = 17$$

46. (a) : Let F be the matrix representing the number of family members and R be the matrix representing the requirement of calories and proteins for each person. Then

$$F = \begin{array}{c} \\ \text{Family } A \\ \text{Family } B \end{array} \begin{array}{c} \begin{array}{ccc} \text{Men} & \text{Women} & \text{Children} \end{array} \\ \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix} \end{array}$$

$$R = \begin{array}{c} \\ \text{Man} \\ \text{Woman} \\ \text{Child} \end{array} \begin{array}{c} \begin{array}{cc} \text{Calories} & \text{Proteins} \end{array} \\ \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix} \end{array}$$

47. (b) : The requirement of calories and proteins for each of the two families is given by the product matrix FR.

$$FR = \begin{bmatrix} 4 & 4 & 4 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 2400 & 45 \\ 1900 & 55 \\ 1800 & 33 \end{bmatrix}$$

$$= \begin{bmatrix} 4(2400+1900+1800) & 4(45+55+33) \\ 2(2400+1900+1800) & 2(45+55+33) \end{bmatrix}$$

$$\begin{array}{cc} \text{Calories} & \text{Proteins} \end{array}$$

$$FR = \begin{bmatrix} 24400 & 532 \\ 12200 & 266 \end{bmatrix} \begin{array}{l} \text{Family } A \\ \text{Family } B \end{array}$$

Thus, requirement of calories of family A is 24400.

48. (c) : The requirement of proteins of family B is 266.

49. (c) : Since, $AB = B$ ...(i) and $BA = A$ ...(ii)

$\therefore \quad A^2 + B^2 = A{\cdot}A + B{\cdot}B$

$= A(BA) + B(AB)$ [Using (i) and (ii)]

$= (AB)A + (BA)B$ [Associative law]

$= BA + AB$ [Using (i) and (ii)]

$= A + B$

50. (a) : $A = (a_{ij})_{m \times n}$, $B = (b_{ij})_{n \times p}$, $C = (c_{ij})_{p \times q}$

$BC = (b_{ij})_{n \times p} \times (c_{ij})_{p \times q} = (d_{ij})_{n \times q}$

$\therefore \quad (BC)A = (d_{ij})_{n \times q} \times (a_{ij})_{m \times n}$

Hence, $(BC)A$ is possible only when $m = q$

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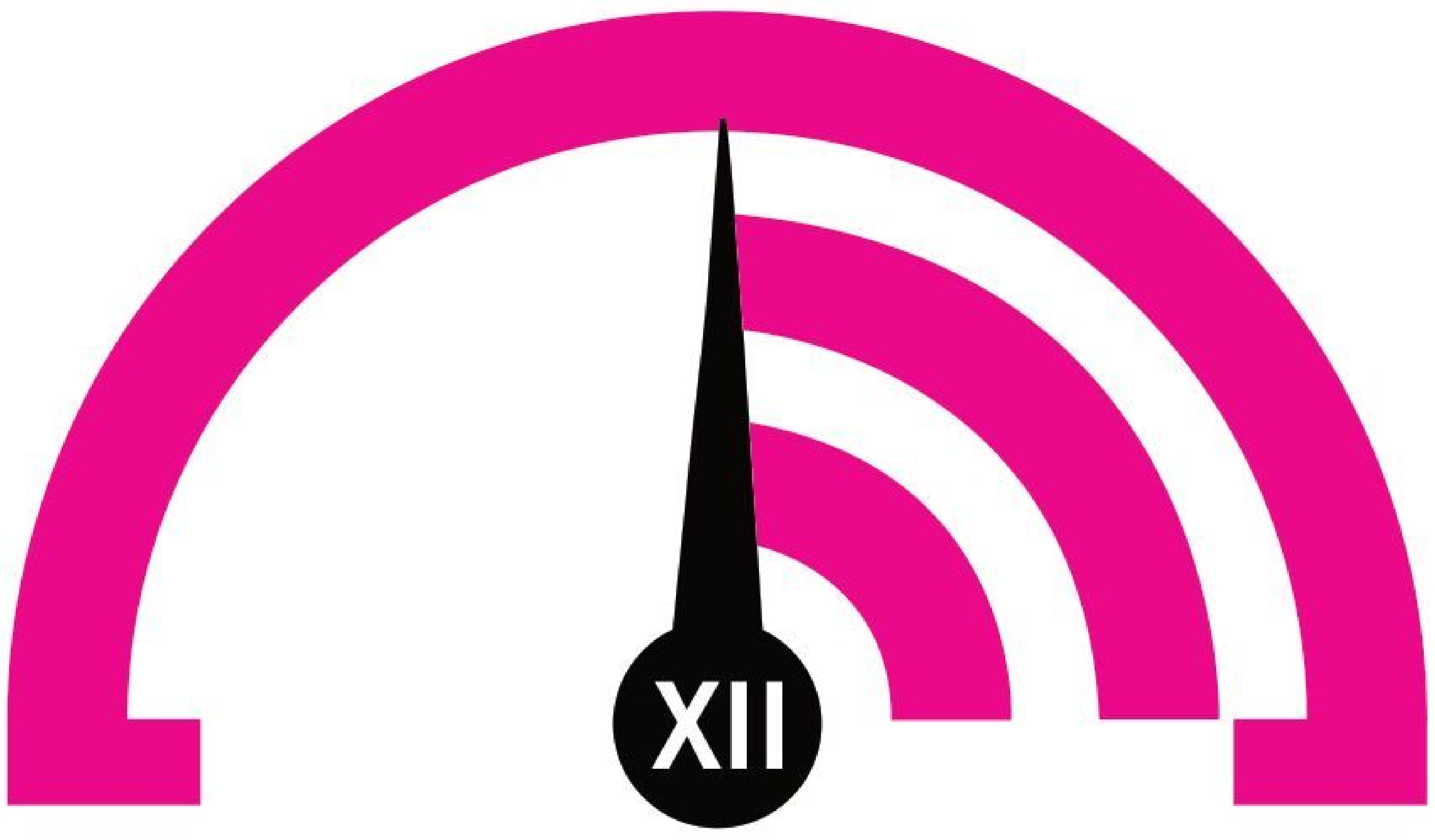

This specially designed column enables students to self analyse their extent of understanding of specified chapters. Give yourself four marks for correct answer and deduct one mark for wrong answer. Self check table given at the end will help you to check your readiness.

Total Marks : 80

Series 4 : Continuity and Differentiability

Time Taken : 60 Min.

Only One Option Correct Type

1. Let $f(x)$ be differentiable function such that $f\left(\dfrac{x+y}{1-xy}\right) = f(x) + f(y)\ \forall\ x$ and y. If $\underset{x\to 0}{\text{Lt}}\ \dfrac{f(x)}{x} = \dfrac{1}{3}$, then $f(1)$ is equal to

(a) $\dfrac{\pi}{4}$ (b) $\dfrac{\pi}{12}$

(c) $\dfrac{\pi}{6}$ (d) None of these

2. Let $f(x) = \begin{cases} \left[\tan\left(\dfrac{\pi}{4}+x\right)\right]^{1/x} &, x \neq 0 \\ k &, x = 0 \end{cases}$, then the value of k such that $f(x)$ is continuous at $x = 0$ is

(a) e (b) $\dfrac{1}{e^2}$

(c) e^2 (d) None of these

3. If $y = f\left(\dfrac{3x+\pi}{5x+3}\right)$ and $f'(x) = \tan^2 x$, then $\dfrac{dy}{dx}$ at $x = 0$ is

(a) $\dfrac{9+5\pi}{3}$ (b) $\dfrac{9-5\pi}{3}$

(c) $\dfrac{5-9\pi}{3}$ (d) $\dfrac{5+9\pi}{3}$

4. For a continuous function f, $f(0) = 0 = f(1)$ and $f'(1) = 2$, $y(x) = f(e^x)\, e^{f(x)}$, then $y'(0)$ equals

(a) 0 (b) 1

(c) 2 (d) None of these

5. If $x^{mx^{mx^{mx^{\cdot^{\cdot^{\cdot^{\infty}}}}}}} = y^{ny^{ny^{ny^{\cdot^{\cdot^{\cdot^{\infty}}}}}}}$, then $\dfrac{dy}{dx}$ equals

(a) $\dfrac{m-n}{m+n}\left(\dfrac{x}{y}\right)$ (b) $\dfrac{m+n}{m-n}\left(\dfrac{y}{x}\right)$

(c) $\dfrac{mx}{ny}$ (d) $\dfrac{my}{nx}$

6. Let $f(x) = \dfrac{3x+2}{5x-3}$, $x \neq \dfrac{3}{5}$, then $\left[\dfrac{d}{dx}\left(f^{-1}(x)\right)\right]_{x=f(1)}$ equals

(a) $-\dfrac{19}{4}$ (b) $-\dfrac{4}{19}$

(c) -19 (d) None of these

One or More than One Option(s) Correct Type

7. If $f(x) = \dfrac{x}{2} - 1$, then on the interval $[0, \pi]$

(a) $\tan f(x)$ and $\dfrac{1}{f(x)}$ are continuous

(b) $\tan f(x)$ and $\dfrac{1}{f(x)}$ are discontinuous

(c) $\tan f(x)$ and $\dfrac{1}{f^{-1}(x)}$ are continuous

(d) $\tan f(x)$ is continuous but $\dfrac{1}{f(x)}$ is not continuous.

8. Let $f(x) = \begin{cases} 1-x, & x<1 \\ (1-x)(2-x), & 1 \le x \le 2 \\ 3-x, & x>2 \end{cases}$. Then $f(x)$ is

(a) continuous, but not differentiable at $x = 1$

(b) differentiable at $x = 1$

(c) continuous, but not differentiable at $x = 2$

(d) discontinuous at $x = 2$

9. Let $f(x) = \begin{cases} -1, & -2 \le x \le 0 \\ x-1, & 0 < x \le 2 \end{cases}$. Then $g(x) = f(|x|) + |f(x)|$ is

(a) differentiable in $(-2, 2)$

(b) differentiable except at $x = 0$

(c) differentiable except at $x = 1$

(d) differentiable except at $x = 0$, $x = 1$

10. If $f(x) = \sec 2x + \text{cosec } 2x$, then $f(x)$ is discontinuous at all points in

(a) $\{n\pi,\ n \in N\}$ (b) $\left\{(2n \pm 1)\frac{\pi}{4}, n \in I\right\}$

(c) $\left\{\frac{n\pi}{4}, n \in I\right\}$ (d) $\left\{(2n \pm 1)\frac{\pi}{8}, n \in I\right\}$

11. If $f(x) = x(\sqrt{x} - \sqrt{x+1})$, then

(a) $f(x)$ is continuous everywhere in the domain
(b) $f(x)$ is differentiable at $x = 0$
(c) $f(x)$ is not differentiable at $x = 0$
(d) None of these

12. If $f(x) = \begin{cases} \dfrac{x \log \cos x}{\log(1+x^2)}, & \text{for } x \neq 0 \\ 0 & \text{, for } x = 0 \end{cases}$, then

(a) $f(x)$ is continuous at $x = 0$
(b) $f(x)$ is continuous but not differentiable at $x = 0$
(c) $f(x)$ is differentiable at $x = 0$
(d) None of these

13. If $f(x) = \begin{cases} \dfrac{x}{1 + e^{\frac{1}{x}}}, & x \neq 0 \\ 0, & x = 0 \end{cases}$, then

(a) $f'(0^+) = 1$ (b) $f'(0^+) = 0$

(c) $f'(0^-) = 1$ (d) $f'(0^-) = 0$

Comprehension Type

Paragraph for Q. No. 14 and 15

If the functions $f(x)$ and $g(x)$ are differentiable in $[a, b]$ and $g'(x) \neq 0$ for $x \in (a, b)$, then there exists c such that $\dfrac{f(b) - f(a)}{g(b) - g(a)} = \dfrac{f'(c)}{g'(c)}$, $a < c < b$.

14. If $f(x) = e^x$, $g(x) = e^{-x}$, then a, b, c are in

(a) A.P. (b) G. P.

(c) H.P. (d) None of these

15. If $f(x) = \sqrt{x}$, $g(x) = \dfrac{1}{\sqrt{x}}$, $a > 0$, then a, b, c are in

(a) A.P. (b) G. P.

(c) H.P. (d) None of these

Matrix Match Type

16. Match the following:

	Column-I	Column-II	
P.	If $y + \sin^2 x = xe^{xy}$, then $y'(0) =$	1.	1
Q.	If $x + \lvert y \rvert = 2y$, then $y'(-1) + y'(1) =$	2.	$\frac{2}{3}$
R.	If $f(x) = \sqrt{x + 2\sqrt{2x-4}} + \sqrt{x - 2\sqrt{2x-4}}$, then $f'(3) + f'(6) =$	3.	$\frac{4}{3}$
S.	If $f(x) = \lvert \sin x - \cos x \rvert$, then $f'\left(\frac{\pi}{2}\right) =$	4.	$\frac{1}{2}$

	P	Q	R	S
(a)	2	1	3	4
(b)	3	4	2	1
(c)	1	3	4	1
(d)	4	2	3	2

Numerical Answer Type

17. If $f(x) = \begin{cases} \dfrac{(5^x - 1)^3}{\sin\left(\frac{x}{a}\right) \log\left(1 + \frac{x^2}{3}\right)}, & x \neq 0 \\ 9(\log 5)^3 & , x = 0 \end{cases}$ is continuous at $x = 0$, then the value of 'a' equals ________.

18. Let $t = \dfrac{2\sqrt{2} - (1 + \sqrt{3})}{\sqrt{3} - 1}$ and $f(x) = \dfrac{2x}{1 - x^2}$, $g(x) = \dfrac{3x - x^3}{1 - 3x^2}$, then $\dfrac{d}{dt}(f(g(t)))$ at t is _______.

19. If $x^3 = \displaystyle\int_0^z \frac{1}{\sqrt{1 + 6t^2}}\, dt$, then $\dfrac{1}{dx/dz}$ equals when $x = 1$, $z = 2$ _______.

20. Let $f_k(x) = \dfrac{1}{k}(\sin^k x + \cos^k x)$, where $x \in R$ and $k \geq 1$, then value of $\dfrac{d}{dx}\left[f_4(x) - f_6(x)\right] + 2$ equals ______.

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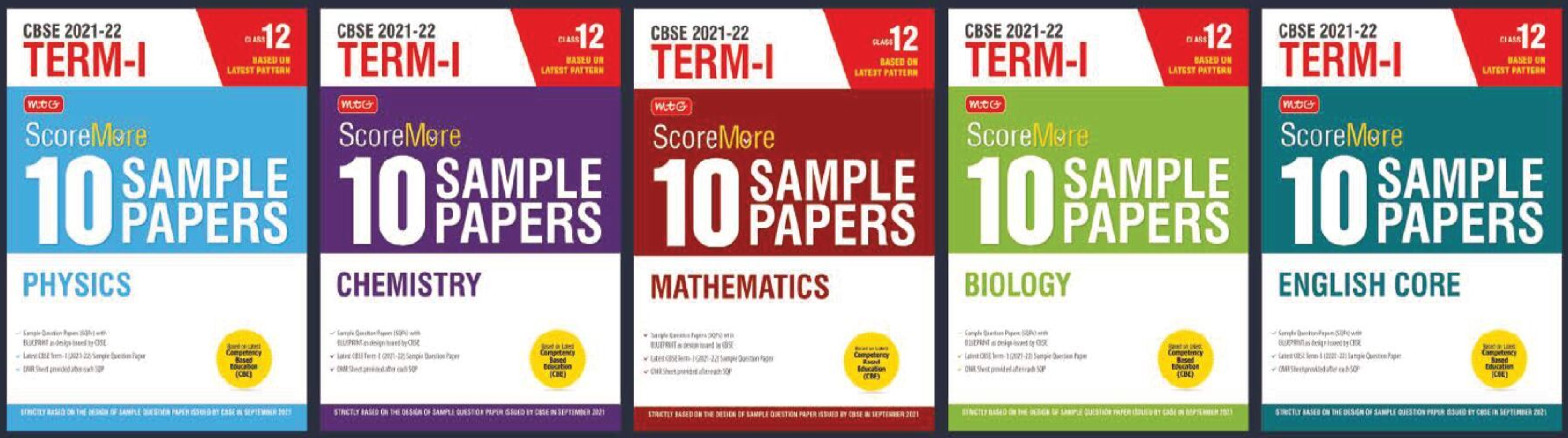
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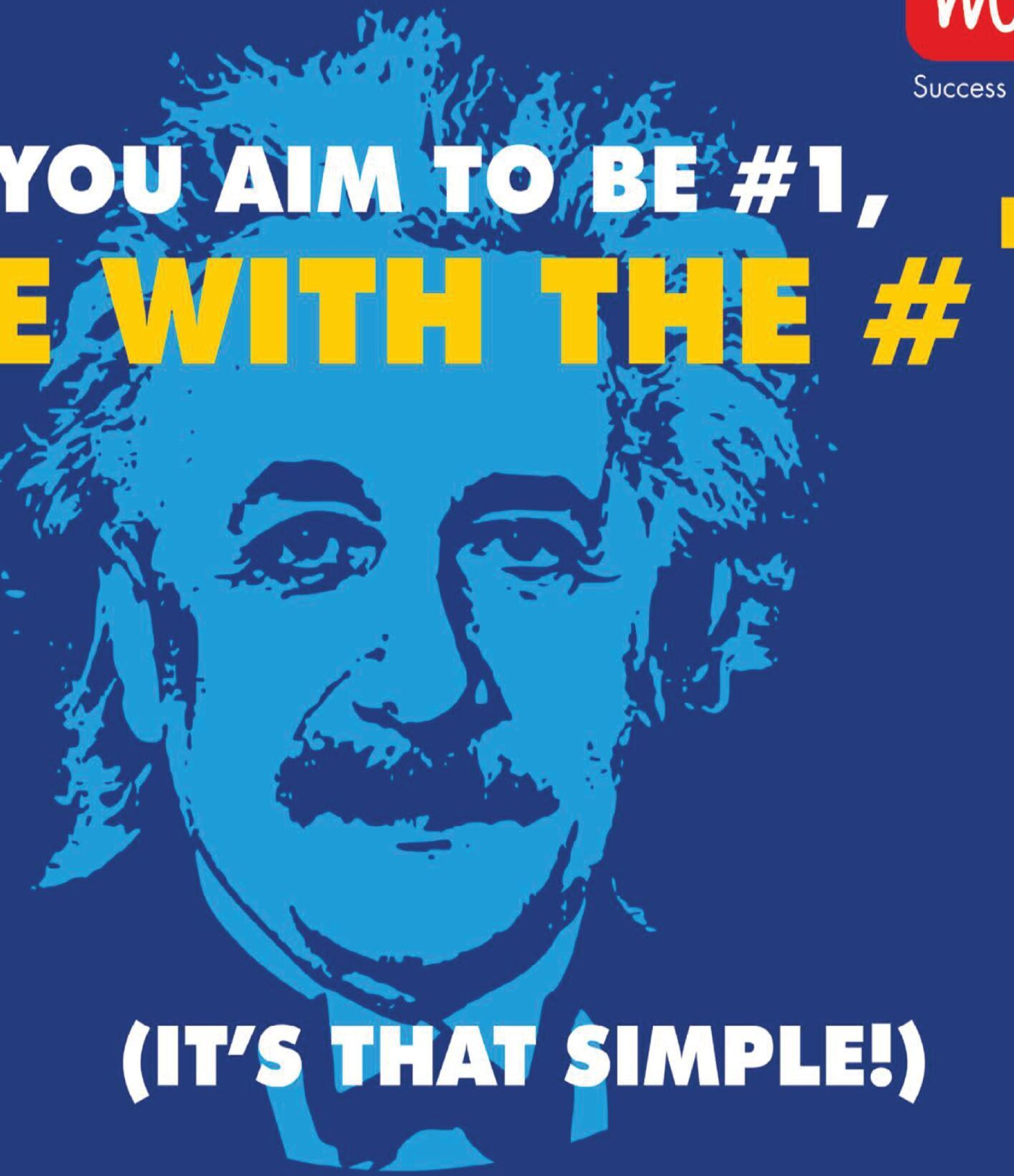

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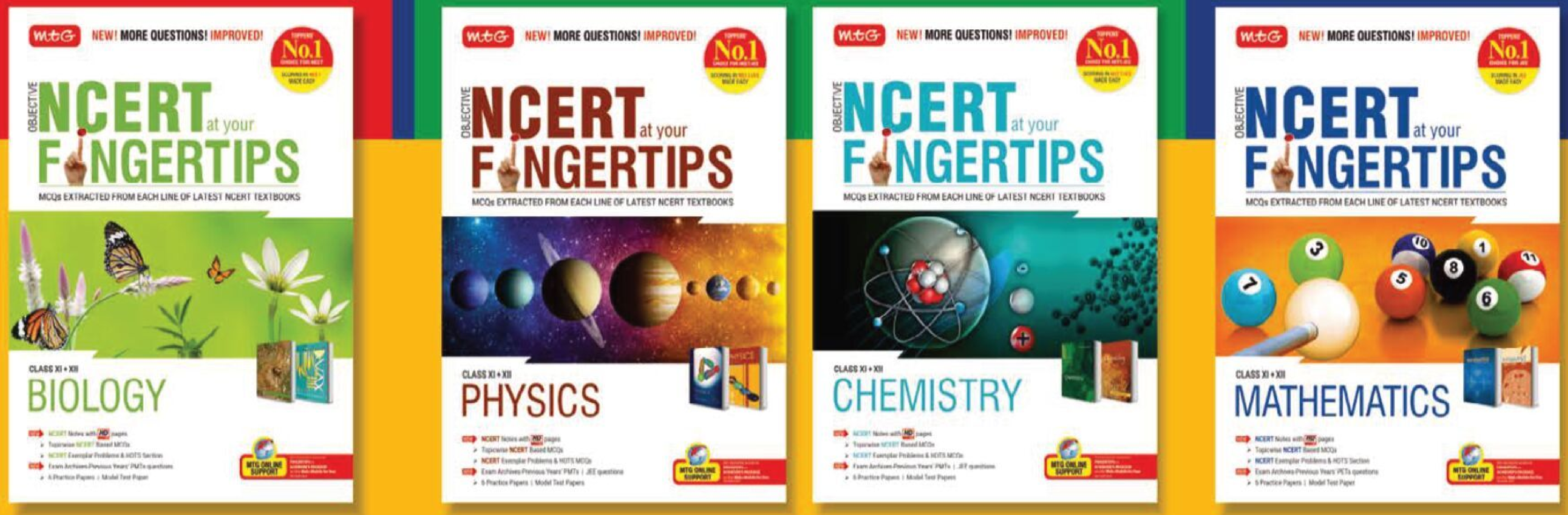
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